Article

Prediction and Analysis of Tides and Tidal Currents

by Jian-Jun SHU, School of Mechanical & Production Engineering, Nanyang Technological University

An efficient algorithm of tidal harmonic analysis and prediction is presented in this paper. The analysis is strengthened by utilising known relationships between tidal constituents found at a neighbouring reference site. The system of linear equations of the least-squares solution is enhanced with included constraint equations. In the case of inadequate data, ill-conditioning in the system of equations that has appeared in other algorithms is conveniently avoided. In solving the resultant normal equations, Goertzel's recurrence formula is adopted so that the whole computation time is dramatically reduced.

List of symbols

A	design matrix	$\vec{Z}, \vec{X}, x, \vec{Y}, y$	unknowns
$B, \bar{B}, R, \bar{R}$	constraint matrices		
C, F, G, S	submatrices	**Greek letters**	
D	constraint relationship	α	amplitude ratio
f	node factor	ΔT	sampling time interval
g	epoch	θ	phase shift
H	mean amplitude	$\vec{\Lambda}, \lambda$	LaGrange multipliers
M	number of tidal constituents	ρ	dummy variable
N	number of observed data	σ	angular velocity
R	amplitude	ϕ	phase
t	time	χ	astronomic argument
$\vec{V}, v$	random errors	**Subscripts**	
$\vec{W}, w$	measured data	j, k, l, m, n	indexes

Tides and tidal currents offer clean and inexhaustible energy sources. Better prediction and analysis of tides and tidal currents are crucial to utilise hydro-dams more efficiently as energy generators. Tides are cyclic variations in the level of seas and oceans, while tidal currents are cyclic variations in the motion of seas and oceans. The present understanding of tides and tidal currents as natural phenomena due to the gravitational forces of the sun and moon acting on a rotating earth came from the development of Newton's gravitation theory [1]. Harmonic techniques were first used to analyse and predict tides and tidal currents by Thomson [2] and expanded by Darwin [3], Harris [4] and Doodson [5]. Tides and tidal currents may be considered as the sum of tidal constituents according to harmonic analysis. With the development of digital computers the least-squares technique is used to evaluate the tidal constituents from observed data and this is a principal method used today. The harmonic method of tidal analysis has been further refined for improvement in accuracy of tidal prediction. A method for superfine resolution of tidal harmonic constituents has been developed by Amin [6-8] adding a corrective step into the harmonic method. The species concordance method has been developed by George & Simon [9] and Simon [10] using relationships between species of the tide at the studied station and at a reference station where the tide is well known or easily predicted. Here we re-examine the harmonic method from a practical point of view and propose an efficient algorithm of tidal harmonic analysis and prediction.

Harmonic Method for Regular Observations

Let us consider real-time regular observed data of tidal height $w_n = w(t_0 + n\Delta T)$ (n= -N, -N +1, ...,N), where t_0 is the mid-point time, $2N$ +1 is the number of the real-time observed data, and ΔT is the sampling time interval. The tidal height can be expressed as a sum of cosine functions plus random errors denoted by $v_n = v(t_0 + n\Delta T)$.

$$w_n = x_0 + \sum_{m=1}^{M} R_m \cos(\phi_m - \sigma_m t_n) + v_n, \qquad n = -N, -N+1, \cdots, N \quad (1)$$

Where

$$t_n = t_0 + n\Delta T, \qquad R_m = f_m H_m, \quad \phi_m = g_m - \chi_m, \qquad m = 1, 2, \cdots, M.$$

$f_m, H_m, \sigma_m, g_m,$ and χ_m are the node factor, mean amplitude, angular velocity, epoch, and astronomical argument of the mth tidal constituent respectively. M is the number of tidal constituents resolved. Eq. (1) can be rewritten as

$$w_n = x_0 + \sum_{m=1}^{M} [x_m \cos(\sigma_m n\Delta T) + y_m \sin(\sigma_m n\Delta T)] + v_n, \qquad n = -N, -N+1, \cdots, N, \quad (2)$$

where

$$\{x_m, y_m\} = R_m [\cos(\phi_m - \sigma_m t_n),\ \sin(\phi_m - \sigma_m t_n)] \qquad m = 1, 2, \cdots, M.$$

Letting $\sigma_0 = 0$ and using matrix notation, Eq. (2) can be expressed as the data equations (observation equations) in matrix-vector form

$$\vec{V} = \vec{W} - A\vec{Z}. \quad (3)$$

The residuals are $\vec{V} = \left(\frac{v_0}{\sqrt{2}}, v_1, \cdots, v_{2N-1}, \frac{v_{2N}}{\sqrt{2}}\right)^T$, the observations are $\vec{W} = \left(\frac{w_0}{\sqrt{2}}, w_1, \cdots, w_{2N-1}, \frac{w_{2N}}{\sqrt{2}}\right)^T$,and the $2M + 1$ unknowns are $\vec{Z}^T = \left(\vec{X}^T, \vec{Y}^T\right)$ where

$$\vec{X}^T = (x_0, x_1, \cdots, x_M), \quad \vec{Y}^T = (y_1, x_2, \cdots, y_M).$$

The column vectors of the measurement matrix (or also termed the \design matrix") A, which is $2N$ +1 rows for the observations $\vec{W}$ and $2M + 1$ columns for the unknowns $\vec{Z}$, are

$$\vec{A}_0, \vec{A}_1, \cdots, \vec{A}_{2M},$$

where

$$\vec{A}_m^T = \left(\frac{1}{\sqrt{2}}\cos[-\sigma_m N\Delta T], \cos[-\sigma_m (N-1)\Delta T], \quad \cdots, \quad \cos[\sigma_m (N-1)\Delta T], \frac{1}{\sqrt{2}}\cos[\sigma_m N\Delta T]\right)$$
$$m = 0, 1, \cdots, M,$$

$$\vec{A}_m^T = \left(\frac{1}{\sqrt{2}}\sin[-\sigma_m N\Delta T], \sin[-\sigma_m (N-1)\Delta T], \quad \cdots, \quad \sin[\sigma_m (N-1)\Delta T], \frac{1}{\sqrt{2}}\sin[\sigma_m N\Delta T]\right)$$
$$m = M+1, M+2, \cdots, 2M.$$

We make the sum of squares of residuals as the mathematical symbol form

$$||\vec{V}||_2^2 = ||\vec{W} - A\vec{Z}||_2^2 = \min,$$

where $||\vec{V}||_2$ is an Euclidean norm (2-norm) of $\vec{V}$. By taking partial derivatives and setting these to zero,

$$\frac{\partial||\vec{V}||_2^2}{\partial x_0} = \frac{\partial||\vec{V}||_2^2}{\partial x_m} = \frac{\partial||\vec{V}||_2^2}{\partial y_m} = 0, \qquad m = 1, 2, \cdots, M,$$

to minimise the 'performance function $||\vec{V}||_2$', also called the objective function, penalty function, or minimand. The derivation yields a set of normal equations

$$A^T A\vec{Z} = A^T\vec{W}. \quad (4)$$

We arrive at

$$A^T A = \begin{bmatrix} F & O \\ O & G \end{bmatrix}, \qquad A^T\vec{W} = \begin{bmatrix} \vec{C} \\ \vec{S} \end{bmatrix},$$

where

$$C_l = \frac{1}{2}\sum_{n=-N+1}^{N} \{w_{n+N-1}\cos[\sigma_l (n-1)\Delta T] + w_{n+N}\cos[\sigma_l n\Delta T]\}, \qquad l = 0, 1, \cdots, M, \quad (5)$$

$$S_l = \frac{1}{2}\sum_{n=-N+1}^{N} \{w_{n+N-1}\sin[\sigma_l (n-1)\Delta T] + w_{n+N}\sin[\sigma_l n\Delta T]\}, \qquad l = 1, 2, \cdots, M. \quad (6)$$

There are analytical expressions, for faster arithmetic,

$$F_{lm} = \frac{1}{2}\left\{\frac{\sin\left[N\left(\sigma_l-\sigma_m\right)\Delta T\right]}{\tan\left[\frac{1}{2}\left(\sigma_l-\sigma_m\right)\Delta T\right]}+\frac{\sin\left[N\left(\sigma_l+\sigma_m\right)\Delta T\right]}{\tan\left[\frac{1}{2}\left(\sigma_l+\sigma_m\right)\Delta T\right]}\right\}, \qquad l,m=0,1,\cdots,M,$$

$$G_{lm} = \frac{1}{2}\left\{\frac{\sin\left[N\left(\sigma_l-\sigma_m\right)\Delta T\right]}{\tan\left[\frac{1}{2}\left(\sigma_l-\sigma_m\right)\Delta T\right]}-\frac{\sin\left[N\left(\sigma_l+\sigma_m\right)\Delta T\right]}{\tan\left[\frac{1}{2}\left(\sigma_l+\sigma_m\right)\Delta T\right]}\right\}, \qquad l,m=1,2,\cdots,M,$$

rather than the usual numerical accumulation of column-vector dot products for F and G, parts of the measurement matrix of the normal equation of the least-squares solution. Please note that by taking limits the answer here for the special case when $l = m$, the first terms appear to come to $2N$ in the analytical expressions for F_{lm} and G_{lm}. Here the formations of the submatrices F and G of the normal equations are derived in the Appendix for clarity. The normal equation Eq. (4) for unknowns x_m parts and unknowns y_m parts is separable and thus can be decomposed into two separate linear equations

$$F\vec{X} = \vec{C}, \qquad G\vec{Y} = \vec{S}. \tag{7}$$

The accuracy of tidal prediction can be improved as longer data time series are analysed and more tidal constituents are selected in Eq. (7).

The assessment of the solution quality can be done by computing $\vec{V}^T\vec{V}$, the sum of squared residuals, as minimised. Then the variance factor is found by $(\vec{V}^T\vec{V})$ /(number of observations - number of unknowns), an estimate of measurement error. In order to examine the solution quality, the covariance matrix of the solution vector of unknowns $\vec{Z}$ can be computed based on the inverse of $A^T A$. The main diagonal values give the standard deviation squared of resolved values of the solution vector $\vec{Z}$, i.e. the accuracy of the resolved constituents, x_m and y_m. From the off-diagonal values, the correlation between resolved constituents can be found. Large values of correlation indicate a weakness in resolving the distinction between tidal constituents.

Goertzel's Recurrence Algorithm for Computing $\vec{C}$ and $\vec{S}$

In terms of complex form

$$C_l + iS_l = \frac{1}{2}\sum_{n=-N+1}^{N}\left\{w_{n+N-1}\exp\left[i\sigma_l\left(n-1\right)\Delta T\right]+w_{n+N}\exp\left[i\sigma_l n\Delta T\right]\right\} \quad l=1,2,\cdots,M. \tag{8}$$

Using Goertzel's recurrence formula [11],

$$\begin{cases}\rho_k = w_k + 2\rho_{k+1}\cos\left(\sigma_l\Delta T\right)-\rho_{k+2} & k=2N-1,2N-2,\cdots,1\\ \rho_0 = \frac{w_0}{2}+2\rho_l\cos\left(\sigma_l\Delta T\right)-\rho_2, & \end{cases} \tag{9}$$

under initial conditions

$$\rho_{2N+1}=0, \quad \rho_{2N}=\frac{w_{2N}}{2}.$$

After $2N$ time recurrences, whence

$$C_l + iS_l = \left[\rho_0 - \rho_1\exp\left(-i\sigma_l\Delta T\right)\right]\exp\left(-i\sigma_l N\Delta T\right). \tag{10}$$

In this method, only $2N$ multiplications are needed. The above take advantage of the equally-spaced data samples of the observed time-series, to yield algorithms with faster arithmetic. Usually these steps are performed by directly number-crunching the matrices.

'Summation of Normals' Method For Segments of Irregular Observations

For K segments of observed data (overlapping is allowed, but not preferred). $w_n^{(k)}$ $(n=-N_k, -N_k+1, \ldots, N_k, k=1,2,\ldots,K)$ are observations with the different length N_k and different sampling time interval ΔT_k For kth segment we have

$$||\vec{V}||_2^2 + \sum_{j=1}^{2J}\lambda_j D_j = \min,$$

where $A_k^T A_k$ is the information matrix, sometimes called the 'Gram matrix'. We add the matrices, for each of the data segments $k = 1$ to K:

$$\sum_{k=1}^{K} A_k{}^T A_k = \text{total}\left(A^T A\right)$$

and the right-hand side vectors:

$$\sum_{k=1}^{K} A_k{}^T \vec{W}_k = \text{total}\left(A^T \vec{W}\right).$$

The information content of each is combined by summing the information into a total information matrix. Although the number of constituents for the kth segment may have a different number M_k, we can set $M = \max_{k=1}^{K} M_k$ for a maximal number of constituents to be chosen for our harmonic analysis. Then the final vector of unknowns $\vec{Z}$ can be found by solving the combined total set of normal equations:

$$\text{total}\left(A^T A\right)\vec{Z} = \text{total}\left(A^T \vec{W}\right).$$

As shown in Eqs. (4) to (7), the normal equations for each data segment are separable. The part for unknowns x_m is independent of the part for y_m. Thus the normal equations can be decomposed into two separate linear equations. This is also true for the combined total set of normal equations found by summation.

Constraints Applied to Strengthen the Solution

In the circumstances of analysed data with insufficient-length (mainly tidal currents), the tidal constituents can not be separated effectively due to ill-conditioning appeared in Eq. (7). Some constraints must be provided. For a pair of tidal constituents, the constraint expressions are as formulated in Dronkers' proposal [12]

$$\begin{bmatrix} D_{2j-1} \\ D_{2j} \end{bmatrix} \equiv \begin{bmatrix} x_{2j-1} \\ y_{2j-1} \end{bmatrix} - \alpha_{2j-1,2j} \begin{bmatrix} \cos\theta_{2j-1,2j} & \sin\theta_{2j-1,2j} \\ -\sin\theta_{2j-1,2j} & \cos\theta_{2j-1,2j} \end{bmatrix} \begin{bmatrix} x_{2j} \\ y_{2j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad j = 1, 2, \cdots, J, \tag{11}$$

where J is the number of tidal constituent pairs, each linked by a constraint relationship to be chosen among the M tidal constituents. As two functions of $\alpha_{2j-1,2j}$is an amplitude ratio and $\theta_{2j-1,2j}$ represents the rotation of a phase shift, which together characterise the relationship between the pair constituents $2j$ - 1 and $2j$. In matrix notation, the constraint equations (11) are most often stated as

$$B\vec{Z} = \vec{O},$$

$$B = \begin{bmatrix} R_1^{(1)} & & & & 0 \; 0 \; \cdots \; 0 & R_1^{(2)} & & & & 0 \; 0 \; \cdots \; 0 \\ & R_2^{(1)} & & & 0 \; 0 \; \cdots \; 0 & & R_2^{(2)} & & & 0 \; 0 \; \cdots \; 0 \\ & & \ddots & & \vdots \; \vdots \; \cdots \; \vdots & & & \ddots & & \vdots \; \vdots \; \cdots \; \vdots \\ & & & R_J^{(1)} & \underbrace{0 \; 0 \; \cdots \; 0}_{(M+1)K-2J} & & & & R_J^{(2)} & \underbrace{0 \; 0 \; \cdots \; 0}_{MK-2J} \end{bmatrix}, \tag{12}$$

where

$$R_j^{(1)} = \begin{bmatrix} 1 & -\alpha_{2j-1,2j}\cos\theta_{2j-1,2j} \\ 0 & \alpha_{2j-1,2j}\sin\theta_{2j-1,2j} \end{bmatrix}, \quad R_j^{(2)} = \begin{bmatrix} 0 & -\alpha_{2j-1,2j}\sin\theta_{2j-1,2j} \\ 1 & -\alpha_{2j-1,2j}\cos\theta_{2j-1,2j} \end{bmatrix}, \quad j = 1, 2, \cdots, J.$$

In the full panel of B, the purpose of the zero-fill elements is to accommodate the tidal constituents not to be chosen in Dronkers' proposal. If taking

$$\|\vec{V}\|_2^2 + \sum_{j=1}^{2J} \lambda_j D_j = \min,$$

and by taking partial derivatives with respect to x_m, y_m and λj and setting these to zero. The derivation yields a set of normal equations

$$\begin{bmatrix} A^TA & B^T \\ B & O \end{bmatrix} \begin{bmatrix} \vec{Z} \\ \vec{\Lambda} \end{bmatrix} = \begin{bmatrix} A^T\vec{W} \\ \vec{O} \end{bmatrix}, \tag{13}$$

where the vector of LaGrange multipliers λj is
$\vec{\Lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_{2J})^T$.

In order to eliminate unknown $\vec{\Lambda}$ in Eq (13), we take

$$\bar{B} = \begin{bmatrix} \bar{R}_1^{(1)} & & & & & & & \bar{R}_1^{(2)} & & & & & & \\ & \bar{R}_2^{(1)} & & & & & & & \bar{R}_2^{(2)} & & & & & \\ & & \ddots & & & & & & & \ddots & & & & \\ & & & \bar{R}_J^{(1)} & & & & & & & \bar{R}_J^{(2)} & & & \\ & & & & 1 & & & & & & & & & \\ & & & & & 1 & & & & & & & & \\ & & & & & & \ddots & & & & & & & \\ & & & & & & & 1 & & & & & & \\ & & & & & & & & & & & 1 & & \\ & & & & & & & & & & & & 1 & \\ & & & & & & & & & & & & & \ddots \\ & & & & & & & & & & & & & & 1 \end{bmatrix}$$

$$\underbrace{\qquad\qquad\qquad}_{(M+1)K} \qquad \underbrace{\qquad\qquad\qquad}_{MK}$$

Where

$$\bar{R}_j^{(1)} = \begin{bmatrix} 1 & \alpha_{2j-1,2j}^{-1}\cos\theta_{2j-1,2j} \\ 0 & -\alpha_{2j-1,2j}^{-1}\sin\theta_{2j-1,2j} \end{bmatrix}, \quad \bar{R}_j^{(2)} = \begin{bmatrix} 0 & \alpha_{2j-1,2j}^{-1}\sin\theta_{2j-1,2j} \\ 1 & \alpha_{2j-1,2j}^{-1}\cos\theta_{2j-1,2j} \end{bmatrix}, \quad j = 1, 2, \cdots, J.$$

The analytic expressions for $\bar{R}_j^{(1)}$ and $\bar{R}_j^{(2)}$ indicate $\bar{R}_j^{(1)} {R_j^{(1)}}^T + \bar{R}_j^{(2)} {R_j^{(2)}}^T = O.$ Then for the composite matrix of constraints B (and $\bar{B}$), we have $\bar{B}BT=0$, i.e., $\bar{B}$ is orthogonal to B. By a simple premultiplication step, the constraints are absorbed into a modified set of normal equations to be solved for the tidal constituents only, and the unwanted vector of LaGrange multipliers is eliminated to reduce the size of the solution.

$$\begin{bmatrix} \bar{B} & O \\ O & I \end{bmatrix} \begin{bmatrix} A^TA & B^T \\ B & O \end{bmatrix} \begin{bmatrix} \vec{Z} \\ \vec{\Lambda} \end{bmatrix} = \begin{bmatrix} \bar{B}A^T\vec{W} \\ \vec{O} \end{bmatrix},$$

where I is unit matrix. Now the rank of matrix equation (13) is reduced by an equivalent form

$$\begin{bmatrix} \bar{B}A^TA \\ B \end{bmatrix} \vec{Z} = \begin{bmatrix} \bar{B}A^T\vec{W} \\ \vec{O} \end{bmatrix}. \tag{14}$$

A novel special method of absorbing the constraints $(B\vec{Z} = \vec{O})$ into the normal equations $(A^TA\vec{Z} = A^T\vec{W})$, and eliminating the unwanted vector of LaGrange multipliers from the solution. The problem of 'ill-conditioning' ('under-determination'), has been pointed here particularly in case of finding harmonic constituents for noisy current observations of short duration. The remedy proposed here for the 'near-singular' solution, is to import a model of the constituents, and bring in their fine structure from a stronger determination at a nearby reference station, and use these in form of constraint equations to strengthen the solution. Mathematically this is done by formulating a 'constrained least squares solution', by applying observations. Constraints are incorporated by the well-known method of LaGrange multipliers. LaGrange multipliers called 'correlates' in the least-squares literature.

Concluding Remarks

The least-squares method has been widely adopted in tidal harmonic analysis. For a concrete problem using a computer, a better algorithm not only requests less computing time but is also able to resolve more effectively the tidal constituents from observed data. An efficient algorithm of tidal harmonic analysis and prediction is presented here. It is the algorithm that can calculate coefficients of normal equations very simply and efficiently. To compute the right-hand terms of the normal equations, Goertzel's recurrence formula [11] is adopted to accomplish the whole calculation processes quickly and accurately. In order to handle the segments of the observed date (mainly adapted to analyse tidal currents), a general algorithm for K sets of real-time irregularly observed date in various observing length can be derived from above results. The 'Summation of Normals' method in which a number K of observed data series are combined in a composite solution. This provides greater exibility in data acquisition and processing. If the above algorithm is used to analyse the tidal constituents, the total analysed data must have sufficient length. Otherwise ill-conditioning in the system of equations appears so that conventional algorithm can not separate tidal constituents effectively. Consequently in the circumstances of inadequate data (mainly for tidal currents), some constraints can be established based on known approximate relationships among the harmonic constants of the tidal constituents. Then the least-squares solutions can be obtained with these constraints applied. To various circumstances, the resultant linear equations can be deduced from this algorithm in order to avoid appropriately the emergence of ill-conditioning. Because the constraints are quite well-defined, the solution usually does not need repeated iterations to converge to sufficient accuracy.

Acknowledgements

The author is grateful to Adam J. Kerr and Nick Stuifbergen and the reviewers for their constructive comments and suggestions.

Appendix

The derivation for F and G can be outlined as follows.

$$\begin{aligned} F_{lm} &= \vec{A}_l^T \vec{A}_m = \sum_{n=0}^{2N-1} \cos\left[(N-n)\,\sigma_l \Delta T\right] \cos\left[(N-n)\,\sigma_m \Delta T\right] \\ &= \frac{1}{2} \sum_{n=0}^{2N-1} \left\{\cos\left[(N-n)\,(\sigma_l-\sigma_m)\,\Delta T\right] + \cos\left[(N-n)\,(\sigma_l+\sigma_m)\,\Delta T\right]\right\}. \end{aligned}$$

Three are two terms in the above equation.
The first term

$$\begin{aligned} \times &\quad \sin\left[\frac{1}{2}(\sigma_l-\sigma_m)\,\Delta T\right] \\ = &\quad \frac{1}{4}\sum_{n=0}^{2N-1}\left\{\sin\left[\left(N-n+\frac{1}{2}\right)(\sigma_l-\sigma_m)\,\Delta T\right] - \sin\left[\left(N-n-\frac{1}{2}\right)(\sigma_l-\sigma_m)\,\Delta T\right]\right\} \\ = &\quad \frac{1}{4}\left\{\sin\left[\left(N+\frac{1}{2}\right)(\sigma_l-\sigma_m)\,\Delta T\right] + \sin\left[\left(N-\frac{1}{2}\right)(\sigma_l-\sigma_m)\,\Delta T\right]\right\} \\ = &\quad \frac{1}{2}\sin\left[N\,(\sigma_l-\sigma_m)\,\Delta T\right]\cos\left[\frac{1}{2}(\sigma_l-\sigma_m)\,\Delta T\right]. \end{aligned}$$

By using the same mathematical manipulation for other terms, the formation of F and G can be expressed analytically.

References

[1] Newton, I (1687). *Philosophia Naturalis Principia Mathematica*

[2] Thomson, W (1868-1876). 'Reports of committee for harmonic analysis', *British Association for the Advancement of Science*

[3] Darwin, GH (1883-1886). 'Reports of a committee for the harmonic analysis of tides', *British Association for the Advancement of Science*

[4] Harris, RA (1897-1907). 'Manual of tides', *Appendices to Reports of the U.S. Coast and Geodetic Survey*

[5] Doodson, AT (1921). 'The harmonic development of the tide-generating potential', *Proceedings of the Royal Society of London Series A. Mathematical and Physical Sciences*, Vol. **100**, pp.305-329

[6] Amin, M (1976). 'The fine resolution of tidal harmonics', *Geophysical Journal of the Royal Astronomical Society*, Vol. **44**, No. 2, pp. 293-310

[7] Amin, M (1987). 'A method for approximating the nodal modulations of the real tide', *International Hydrographic Review*, Vol. **64**, No.2, pp. 103-113

[8] Amin, M (1991). 'Superfine resolution of tidal harmonic constants', in: *Tidal Hydrodynamics*, BB Parker, ed, John Wiley & Sons, Inc., pp. 711-724

[9] George, KJ & Simon, B (1984). 'The species concordance method of tide prediction in estuaries', *International Hydrographic Review*, Vol. **65**, No.1 pp.121-146

[10] Simon, B (1991). 'The species concordance method of tide prediction', in: *Tidal Hydrodynamics*, BB Parker, ed, John Wiley & Sons, Inc., pp. 725-735

[11] Goertzel, G (1958). 'An algorithm for the evaluation of finite trigonometric series', *The American Mathematical Monthly*, Vol. **65**, No.1, pp. 34-35

[12] Dronkers, JJ (1964). *Tidal computations in rivers and coastal waters*, Amsterdam: North-Holland Publishing Co

Biography

Dr. Jian-Jun SHU is a recipient of the British Institution of Mechanical Engineers 1992 BFPA Prize for Young Engineers. He has published over 60 technical papers and presented over 20 invited lectures/seminars. He is with Nanyang Technological University, Singapore as an Associate Professor.

E-mail: MJJShu@ntu.edu.sg