



# Screw dislocation interacting with imperfect interface

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## Abstract

The study of dislocations interacting with interfaces is of great interest to understand the strengthening and hardening mechanisms in materials. Most of the existing results in this field dealt with perfect interfaces where the displacements are continuous across interfaces. In reality, however, due to damage of interfaces or other reasons, the displacements across interfaces could have a jump. In the present study, a linear spring model is applied to study imperfect interfaces, and the interaction of a screw dislocation with an imperfect interface is formulated. The results show that the interacting force on a screw dislocation with an imperfect interface varies between that with a free surface and that with a perfect interface. It should be pointed out that the present study is the first step of the dislocation/imperfect-interface research. The interaction between edge dislocations and imperfect interface is a more practically important and challenging problem.

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## 1. Introduction

The analytical research on the interaction between interfaces and dislocations started from early 1950s by Head (1953) who analyzed force on a screw dislocation near an interface of a bi-material. Since then dislocation interacting with interfaces has been an active research topic for solid mechanics researchers and material scientists. A thorough review on this topic would lead to a very lengthy introduction, which cannot be fitted into the present paper. Fortunately, the following

two review articles give us a reasonably clear picture of evolution of the dislocation/interface interaction research. The first detailed review article was written by Dundurs (1969), where he summarized the major contributions to the topic up to the end of 1960s. Most recently, Chen (2001) reviewed the progress of the interaction research in the past thirty years, as part of his effort to study dislocations interacting with wedged interfaces and inhomogeneities. Since the solution for a single dislocation interacting with interface is considered as a Green's function, there have been a number of research topics derived from this fundamental solution. For example, the cracks (Griffith crack and Zener–Stroh crack) can be formulated by using dislocation pileup concept (Weertman, 1996). The

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plasticity and strengthening phenomena have also been explained and modeled by the dislocation mechanisms (Hirth and Lothe, 1982). Therefore, the research on the dislocation/interface interaction is not only important to the dislocation researchers, but also to the fracture mechanics, plasticity and composite mechanics researchers, to name a few.

It is noticed that early research works (before 1980s) on dislocation/interface interaction treated the interface as perfectly bonded. In mechanics terminology, a perfect interface is described by, referring to the configuration of Fig. 1(a),

$$T_i^{(2)}(0, y) = T_i^{(1)}(0, y), \quad \text{traction continuity,}$$

and

$$u_i^{(1)}(0, y) = u_i^{(2)}(0, y), \quad \text{displacement continuity.}$$

The superscripts are used to distinguish between the two materials.

It was realized that there might be a jump of displacements across the interface due to damage or imperfection of the interface. Since the 1980s, imperfect interfaces have been attracting more and more attention. Among the numerous models for the imperfect interface, the linear spring model has been widely employed due to its simplicity and feasibility in applications. Again, in mechanics terminology, the imperfect interface described by the linear spring model is given by

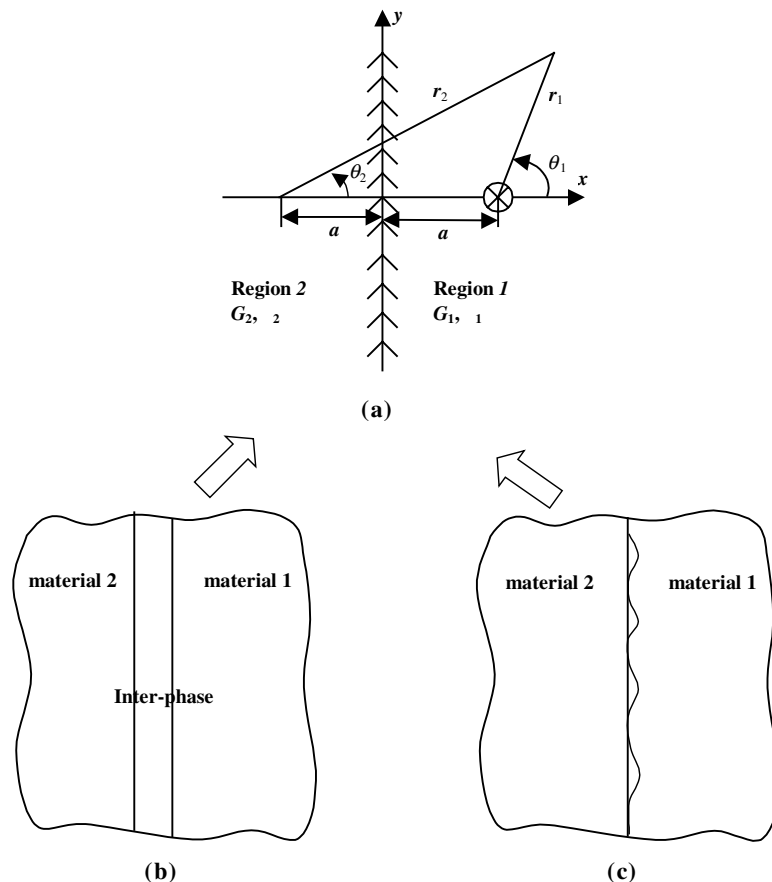


Fig. 1. (a) A screw dislocation near interface, (b) inter-phase modeled by spring model, (c) a damaged interface modeled by spring model.

$$\sigma_{xz}^{(2)}(0, y) = \sigma_{xz}^{(1)}(0, y)$$

and

$$k(u_z^{(1)}(0, y) - u_z^{(2)}(0, y)) = \sigma_{xz}^{(1)}(0, y)$$

for the anti-plane problem shown in Fig. 1(a), where  $k$  is the so-called spring constant. There are few research works for the interaction between dislocations and imperfect interfaces with various interface models. Stagni and Lizzio (1992) investigated the dislocation in a lamella inhomogeneity with slipping interfaces. Chen et al. (1998) considered the dislocation near a sliding interface. Shilkrot and Srolovitz (1998) studied a dislocation interacting with an interface with a finite stiffness.

There are many research works for imperfect interface, but not for the interaction between dislocations and the imperfect interface. On one hand, experimentalists (Margetan et al., 1988; Lavrentyev and Rokhlin, 1998) showed that spring model was suitable for simulating certain kind of interfaces. On the other hand, many analytical works have also been carried out with the spring model. For example, Hashin (1990, 1991) derived the spherical inclusion solution with imperfect interface, where he explained that the spring model could be viewed as inter-phase as shown in Fig. 1(b). Fan and Sze (2001) related the micro-cracks on the interface with the spring constant of the interface by using a self-consistent scheme as shown in Fig. 1(c). Recently, Zhong and Meguid (1997) reviewed the research works on the imperfect interface and its application in composite mechanics. Since the imperfect interface is not limited to solid mechanics problems, there are also some works for other physical problems. For example, Benveniste (1999) considered an imperfect interface in conduction phenomena.

In the following sections, we will focus on the interaction between a screw dislocation and imperfect interfaces described by a linear spring model. In terms of geometrical configurations, we will consider (1) two joint infinitely extended half-spaces, (2) one half space coated with a thin layer, and (3) a circular inhomogeneity embedded in an infinite matrix. It is desirable that the solutions for all these configurations are simple enough to be

applied as fundamental solutions for other applications.

We would like to point out that the present anti-plane problem is the simplest configuration among all dislocation/interface problems. Plane problems may encounter bigger mathematical difficulties and reveal more physical phenomena. This paper is considered as the first step towards a thorough understanding of dislocation interacting with imperfect interfaces.

## 2. A screw dislocation near an imperfect interface

As our first geometric configuration, we consider a screw dislocation near an imperfect interface as depicted in Fig. 1. The coordinates are set up in such a way that the interface is along the  $y$ -axis and the screw dislocation is located in material 1 at point  $(a, 0)$ . Materials 1 and 2 are linear elastic with shear moduli  $G_1$  and  $G_2$ , respectively.

For the anti-plane problem, the only non-vanishing displacement,  $u_z$ , is the function of coordinates  $x$  and  $y$  only, and satisfies Laplace's equation

$$\nabla^2 u_z = 0. \quad (1)$$

The non-vanishing stress components are given by Hooke's law,

$$\sigma_{xz} = G \frac{\partial u_z}{\partial x} \quad \text{and} \quad \sigma_{yz} = G \frac{\partial u_z}{\partial y}. \quad (2)$$

Although any plane with the screw dislocation could be taken as the slip plane, it is convenient to select the plane  $y = 0$  as the slipping plane. Therefore, the dislocation condition can be expressed as

$$\lim_{\eta \rightarrow 0} [u_z(x, -\eta) - u_z(x, \eta)] = b \quad (\text{for } \eta > 0 \text{ and } x > a), \quad (3)$$

where  $b$  is the magnitude of Burgers' vector. For the present imperfect interface problem, the traction across the interface is continuous, that is

$$\sigma_{xz}^{(2)}(0, y) = \sigma_{xz}^{(1)}(0, y). \quad (4)$$

As to the displacement condition, the spring model is adopted here, with which the jump of displacement  $u_z$  across interface is linearly proportional to the traction  $\sigma_{xz}$ ,

$$k(u_z^{(1)}(0, y) - u_z^{(2)}(0, y)) = \sigma_{xz}^{(1)}(0, y) \tag{5}$$

where  $k$  is the spring constant of the interface. The interface is perfectly bonded for  $k$  approaching infinity; while the interface is a free surface for  $k$  approaching zero.

The solution to this boundary value problem, Eqs. (1)–(5), can be expressed as

$$\begin{aligned} u_z^{(1)} &= \frac{b}{2\pi} \underline{(\theta_1 + K\theta_2)} + \hat{u}_z^{(1)}, \\ u_z^{(2)} &= \frac{b}{2\pi} \underline{[(1 - K)\theta_1 + K\pi]} + \hat{u}_z^{(2)}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} K &= \frac{\Gamma - 1}{\Gamma + 1}, \\ \Gamma &= G_2/G_1, \end{aligned} \tag{7}$$

and

$$0 \leq \theta_1, \theta_2 \leq 2\pi.$$

In Eq. (6), the underlined terms are the solutions for the perfect interface (Dundurs, 1969), while  $\hat{u}_z^{(1)}$  and  $\hat{u}_z^{(2)}$  collect the effect of imperfect interface. It is obvious that  $\hat{u}_z^{(1)}$  and  $\hat{u}_z^{(2)}$  are also harmonic functions, namely,

$$\nabla^2 \hat{u}_z^{(1)} = 0 \quad \text{and} \quad \nabla^2 \hat{u}_z^{(2)} = 0. \tag{8}$$

By applying Fourier transformation,

$$U_z(x, s) = \int_{-\infty}^{\infty} u_z(x, y) e^{-isy} dy, \tag{9}$$

$$u_z(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U_z(x, s) e^{isy} ds \tag{10}$$

to Eq. (8), we obtain

$$\left( \frac{d^2}{dx^2} - s^2 \right) U_z(x, s) = 0. \tag{11}$$

Since the displacement is finite as  $x \rightarrow \infty$ , the Fourier transformations of the displacements  $\hat{u}_z^{(1)}$  and  $\hat{u}_z^{(2)}$  should take the forms

$$\hat{U}_z^{(1)} = A(s) e^{-|s|(x-a)}, \tag{12}$$

$$\hat{U}_z^{(2)} = C(s) e^{|s|(x+a)}. \tag{13}$$

By using Eqs. (4) and (5),  $A(s)$  and  $C(s)$  can be determined by

$$A(s) = -\Gamma C(s), \tag{14}$$

$$C(s) = \frac{i}{2} b(K - 1) \operatorname{sgn}(s) \frac{ae^{-2a|s|}}{a|s| + \lambda}, \tag{15}$$

where  $\operatorname{sgn}(s)$  is the sign function

$$\operatorname{sgn}(s) = \begin{cases} 1, & s > 0, \\ 0, & s = 0, \\ -1, & s < 0, \end{cases} \tag{16}$$

and

$$\lambda = ak \frac{G_1 + G_2}{G_1 G_2}. \tag{17}$$

Parameter  $\lambda$  is a dimensionless constant, which measures the interface “rigidity”. It will be seen that the parameter  $\lambda$  is the only new parameter introduced into all the solutions for the imperfect interface configurations.

Substituting of Eqs. (14) and (15) into Eqs. (12) and (13), the displacements  $\hat{u}_z^{(1)}$  and  $\hat{u}_z^{(2)}$  can be expressed by

$$\hat{u}_z^{(1)}(x, y) = \frac{b\Gamma(K - 1)}{2\pi} \int_0^{\infty} \frac{ae^{-s(x+a)}}{as + \lambda} \sin(sy) ds, \tag{18}$$

$$\hat{u}_z^{(2)}(x, y) = -\frac{b(K - 1)}{2\pi} \int_0^{\infty} \frac{ae^{s(x-a)}}{as + \lambda} \sin(sy) ds. \tag{19}$$

With Hooke’s law, Eq. (2), the total stresses in materials 1 and 2 can be written as

$$\begin{aligned} \sigma_{zx}^{(1)}(x, y) &= \frac{G_2 b(K - 1)}{2\pi} \int_0^{\infty} \frac{\lambda \sin(sy)}{as + \lambda} e^{-s(x+a)} ds \\ &\quad - \frac{bG_1}{2\pi} \left( \frac{y}{r_1^2} - \frac{y}{r_2^2} \right), \end{aligned} \tag{20}$$

$$\begin{aligned} \sigma_{zy}^{(1)}(x, y) &= -\frac{G_2 b(K - 1)}{2\pi} \int_0^{\infty} \frac{\lambda \cos(sy)}{as + \lambda} e^{-s(x+a)} ds \\ &\quad + \frac{bG_1}{2\pi} \left( \frac{x - a}{r_1^2} - \frac{x + a}{r_2^2} \right), \end{aligned} \tag{21}$$

$$\sigma_{zx}^{(2)}(x, y) = \frac{G_2 b(K - 1)}{2\pi} \int_0^{\infty} \frac{\lambda \sin(sy)}{as + \lambda} e^{s(x-a)} ds, \tag{22}$$

$$\sigma_{zy}^{(2)}(x, y) = \frac{G_2 b(K-1)}{2\pi} \int_0^\infty \frac{\lambda \cos(sy)}{as + \lambda} e^{s(x-a)} ds. \tag{23}$$

In Eqs. (20) and (21), the underlined terms are the results of free surface, while the other terms are the effects of the interface.

The elastic energy can then be calculated as

$$\begin{aligned} E &= \frac{b}{2} \int_{a+r_0}^R \sigma_{zy}(x, 0) dx \\ &= \frac{b^2 G_1}{4\pi} \ln\left(\frac{2a}{r_0}\right) - \frac{b^2(K-1)G_2}{4\pi} \\ &\quad \times \int_{a+r_0}^R \int_0^\infty \frac{\lambda e^{-s(x+a)}}{as + \lambda} ds dx \\ &= \frac{b^2 G_1}{4\pi} \ln\left(\frac{2a}{r_0}\right) - \frac{b^2(K-1)G_2}{4\pi} \\ &\quad \times \int_0^\infty \frac{\lambda}{as + \lambda} \left[ \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right] ds, \end{aligned} \tag{24}$$

where  $R \gg a$  and  $r_0$  is the radius of dislocation core used to eliminate the impropriety of elastic theory in the core. The interchange of integration order in Eq. (24) is discussed in detail in Appendices A, B and C.

Furthermore, the force acting on the dislocation is given by

$$F_x = -\frac{\partial E}{\partial a} = -\frac{b^2 G_1}{4\pi a} \left[ 1 - \frac{2\Gamma}{\Gamma+1} f(\lambda) \right], \tag{25}$$

where

$$f(\lambda) = 2\lambda e^{2\lambda} \text{Ei}(2\lambda), \tag{26}$$

with

$$\text{Ei}(2\lambda) = \int_{2\lambda}^\infty \frac{e^{-t}}{t} dt. \tag{27}$$

Sequence interchange of differentiation and integration in Eq. (25) is also discussed in detail in Appendices A, B and C.

Fig. 2 shows the monotonic variation of  $f(\lambda)$  with respect to  $\lambda$ . The value of  $f(\lambda)$  falls into the range between 0 and 1. As  $\lambda$  approaches infinity, the interface becomes perfectly bonded, and  $f(\lambda)$  equals to 1. Therefore, the results given by Eq. (25) reduce to that for the perfect interface (Head,

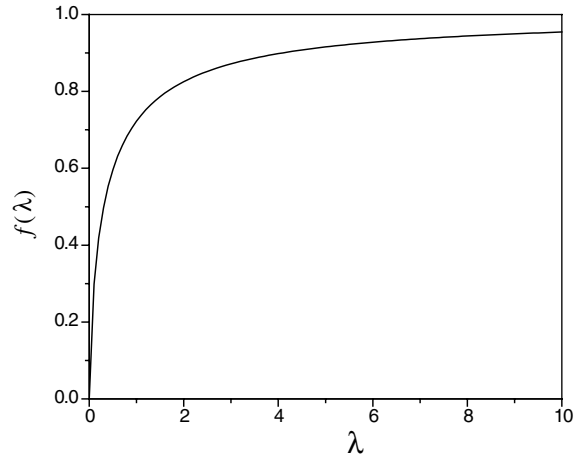


Fig. 2. The variation of  $f(\lambda)$  with respect to  $\lambda$ .

1953). As  $\lambda$  approaches zero, the interface becomes a free surface, and  $f(\lambda)$  equals zero. Therefore, the effects of imperfect interface on a screw dislocation is completely determined by the dimensionless parameter  $\lambda$ , which is related to the spring constant, the shear moduli of two materials and the location of dislocation.

### 3. Screw dislocation near surface layer

In addition to the interface, surface layers are also encountered in engineering applications. Based on the solutions of Eqs. (20)–(23), we are able to deal with the effects of surface layer on a screw dislocation as depicted in Fig. 3. The boundary value problem is reduced to finding an additional solution to Eq. (6), which can clear the traction on the free surface  $x = -h$  from Eq. (22).

Following a similar procedure as in Section 2, we can obtain Fourier transformation of the displacement as

$$\widehat{U}_z^{(1)} = A(s)e^{-|s|x}, \tag{28}$$

$$\widehat{U}_z^{(2)} = B(s) \sinh(sx) + C(s) \cosh(sx). \tag{29}$$

Using the free surface condition at  $x = -h$  and the interface condition at  $x = 0$  given by Eqs. (4) and (5), we obtain

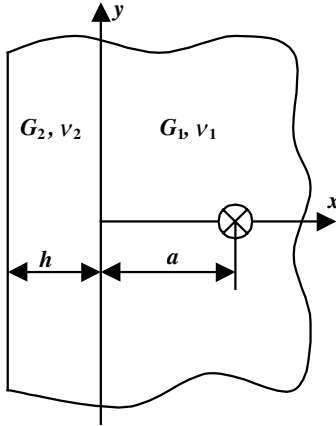


Fig. 3. A screw dislocation near a surface layer.

$$A(s) = -\Gamma \frac{s}{|s|} B(s),$$

$$C(s) = -\left( \frac{1+\Gamma}{\lambda} as + \Gamma \frac{s}{|s|} \right) B(s), \tag{30}$$

$$B(s) = b(K-1) \frac{i}{2} \frac{\lambda e^{-s(h+a)}}{as^2 + \lambda|s|} \left[ \cosh(sh) + \left( \frac{1+\Gamma}{\lambda} a + \frac{\Gamma}{|s|} \right) s \sinh(sh) \right]^{-1}. \tag{31}$$

Accordingly, the displacement and then the stress components can be determined. The elastic energy is given by

$$E = (K-1) \frac{G_2 b^2}{4\pi} \times \int_0^\infty \frac{\lambda e^{-s(h+a)}}{as + \lambda} \left[ \frac{e^{-(a+r_0)s} - e^{-Rs}}{s} \right] \left[ \cosh(sh) + \left( \frac{1+\Gamma}{\lambda} as + \Gamma \right) \sinh(sh) \right]^{-1} ds. \tag{32}$$

Again, there is a procedure of interchange of integration order in Eq. (32), which can be discussed by using a similar proof as provided in Appendices A, B and C.

With the elastic energy, we can calculate the total force acting on dislocation as

$$F_x = -\frac{\partial E}{\partial a} = -\frac{b^2 G_1}{4\pi a} \left[ 1 - \frac{2\Gamma}{\Gamma+1} g(h/a, \lambda, \Gamma) \right], \tag{33}$$

where

$$g(h/a, \lambda, \Gamma) = f(\lambda) - \int_0^\infty \frac{2\lambda e^{-2t}}{t + \lambda} \times \frac{\exp(-\frac{h}{a}t)}{\cosh(\frac{h}{a}t) + (\frac{1+\Gamma}{\lambda}t + \Gamma) \sinh(\frac{h}{a}t)} dt \tag{34}$$

with  $f(\lambda)$  defined by Eq. (26).

It is found from Eqs. (33) and (34) that when  $h = 0$ ,  $g(0, \lambda, \Gamma)$  equals zero, the force acting on dislocation reduces to that of free surface; while when  $h$  tends to infinity,  $g(h/a, \lambda, \Gamma)$  equals to  $f(\lambda)$ , Eq. (34) reduces to Eq. (25). When  $\lambda$  approaches infinity, Eq. (33) reduces to the results for

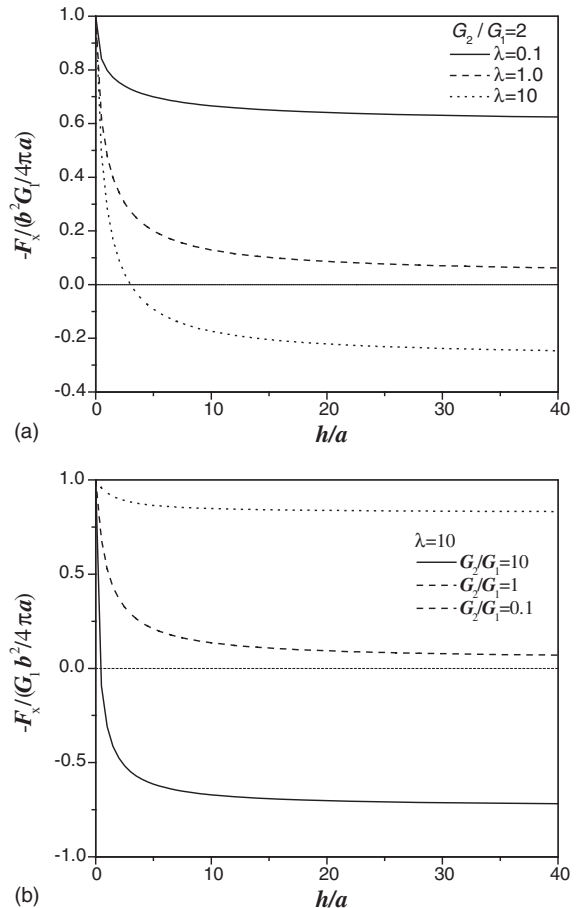


Fig. 4. (a) The variation of normalized  $F_x$  (Eq. (33)) vs.  $h/a$  for various  $\lambda$ . (b) The variation of normalized  $F_x$  (Eq. (33)) vs.  $h/a$  for various  $\Gamma$ .

a perfect interface (Dundurs, 1969). Fig. 4(a) shows the monotonic variation of normalized force  $F_x$  with respect to  $h/a$  for a fixed  $\Gamma = 2$  but with various  $\lambda$ . It is known that the dislocation has an equilibrium position (zero force) for  $\Gamma > 0$  and perfect interface. This phenomenon indicates the competition between the attractive force exerted on the dislocation by the free surface  $x = -h$  and repulsive force exerted by the layer with higher modulus. But this equilibrium position may disappear as the bonding of the interface deteriorates ( $\lambda$  decreases). Fig. 4(b) shows the variation of the normalized force verse  $h/a$  for a fixed  $\lambda = 10$  but with various  $\Gamma$ .

**4. Screw dislocations interacting with a circular inclusion**

Firstly, let us consider a screw dislocation in the matrix as shown in Fig. 5, where  $\xi > 1$ . Laplace’s equation takes the following form in a polar coordinate:

$$\nabla^2 u_z = \left( \frac{\partial^2}{\partial r^2} + \frac{\partial}{r \partial r} + \frac{\partial^2}{r^2 \partial \theta^2} \right) u_z = 0. \tag{35}$$

Its general solution can be given as

$$u_z^{(1)} = \frac{b}{2\pi} \left\{ \frac{\theta_1 + K(\theta_2 - \theta_1)}{1} + \sum_{n=1}^{\infty} \left( \frac{a}{r} \right)^n [a_n \cos(n\theta) + b_n \sin(n\theta)] \right\}, \tag{36}$$

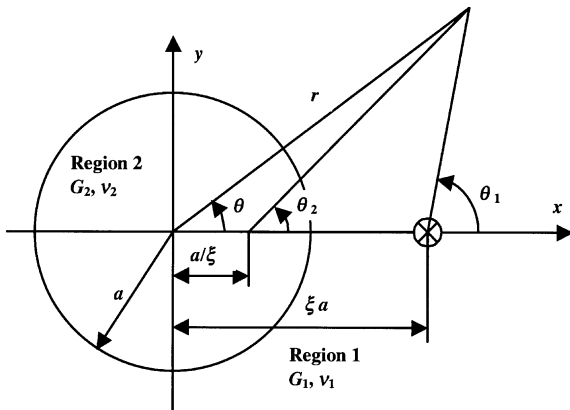


Fig. 5. A screw dislocation near a circular inclusion.

$$u_z^{(2)} = \frac{b}{2\pi} \left\{ \frac{(1-K)\theta_1 + \pi K}{1} + \sum_{n=1}^{\infty} \left( \frac{r}{a} \right)^n [c_n \cos(n\theta) + d_n \sin(n\theta)] \right\}, \tag{37}$$

where  $0 \leq \theta_1, \theta_2 \leq 2\pi$  and  $K$  is defined by Eq. (7). Again, the underlined terms in Eqs. (36) and (37) are the solutions for the perfect interface (Dundurs, 1969). The effect of the imperfection of the interface is taken into account by the series solution. The boundary conditions on the interface are given by

$$k(u_z^{(1)}(a, \theta) - u_z^{(2)}(a, \theta)) = \sigma_{rz}^{(1)}(a, \theta) \tag{38}$$

and

$$\sigma_{rz}^{(2)}(a, \theta) = \sigma_{rz}^{(1)}(a, \theta),$$

where

$$\sigma_{rz} = \sigma_{yz} \sin \theta + \sigma_{xz} \cos \theta. \tag{39}$$

Substituting Eqs. (36) and (37) into Eq. (38) leads to

$$a_n = c_n = 0, \quad b_n = -\Gamma d_n, \tag{40}$$

$$d_n = \left( \frac{1}{n + \lambda} \right) \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{(1-K)\xi \sin \theta}{1 + \xi^2 - 2\xi \cos \theta} \times \sin(n\theta) d\theta, \tag{41}$$

where  $\lambda$  is defined in Eq. (17).

The interaction energy of dislocation can be expressed by

$$E = \frac{b^2 G_1}{4\pi} \left[ K \log \left( \frac{\xi^2}{\xi^2 - 1} \right) + \Gamma(K - 1) \times \sum_{n=1}^{\infty} \left( \frac{1}{n + \lambda} \right) \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{\xi^{1-n} \sin \theta \sin(n\theta)}{1 + \xi^2 - 2\xi \cos \theta} d\theta \right], \tag{42}$$

and the force on the dislocation is calculated as

$$F_x = \frac{b^2 G_1 K}{2\pi a \xi (\xi^2 - 1)} \left[ 1 + \frac{\Gamma}{\Gamma - 1} g_1(\lambda, \xi) \right], \tag{43}$$

where

$$g_1(\lambda, \xi) = \xi(\xi^2 - 1) \sum_{n=1}^{\infty} \left( \frac{\xi^{-n}}{n + \lambda} \frac{1}{\pi} \times \int_{-\pi}^{\pi} \frac{1 - n - (1 + n)\xi^2 + 2n\xi \cos \theta}{(1 + \xi^2 - 2\xi \cos \theta)^2} \times \sin \theta \sin(n\theta) d\theta \right). \quad (44)$$

Validity of order interchange of integration and differentiation in Eqs. (42) and (43) can be proved via the theorems in Appendices A, B and C.

Secondly, as the counterpart problem of the above configuration, let us consider the case that the dislocation is inside the inclusion,  $\xi < 1$ . The displacement field is specified by

$$u_z^{(1)} = \frac{b}{2\pi} \left[ (1 + K)\theta_1 - K\theta + \sum_{n=1}^{\infty} \left( \frac{a}{r} \right)^n (a_n \cos(n\theta) + b_n \sin(n\theta)) \right], \quad (45)$$

$$u_z^{(2)} = \frac{b}{2\pi} \left[ \theta_1 + K(\pi - \theta_2) + \sum_{n=1}^{\infty} \left( \frac{r}{a} \right)^n (c_n \cos(n\theta) + d_n \sin(n\theta)) \right], \quad (46)$$

where the relations among  $a_n, b_n, c_n$  and  $d_n$  are the same as those in Eqs. (40) and (41). The energy of configuration is calculated as

$$E = \frac{b^2 G_2}{4\pi} \left[ K \log(1 - \xi^2) - \frac{2}{\Gamma + 1} \times \sum_{n=1}^{\infty} \left( \frac{1}{n + \lambda} \right) \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{\xi^{n+1} \sin \theta \sin(n\theta)}{1 + \xi^2 - 2\xi \cos \theta} d\theta \right], \quad (47)$$

and the force on the dislocation is

$$F = \frac{b^2 G_2}{2\pi a} \frac{K\xi}{1 - \xi^2} \left[ 1 + \frac{1}{\Gamma - 1} g_2(\lambda, \xi) \right], \quad (48)$$

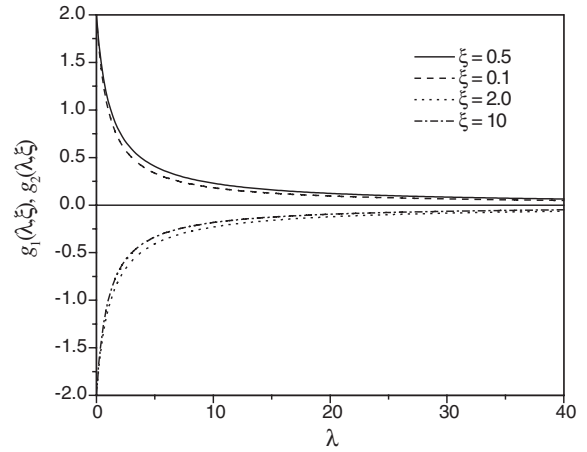


Fig. 6. The variation of  $g_1(\xi, \lambda)$  and  $g_2(\xi, \lambda)$  with respect to  $\lambda$ .

where

$$g_2(\lambda, \xi) = \frac{1 - \xi^2}{\xi} \sum_{n=1}^{\infty} \frac{\xi^n}{n + \lambda} \frac{1}{\pi} \times \int_{-\pi}^{\pi} \frac{n + 1 + (n - 1)\xi^2 - 2n\xi \cos \theta}{(1 + \xi^2 - 2\xi \cos \theta)^2} \times \sin \theta \sin(n\theta) d\theta. \quad (49)$$

Fig. 6 shows the variation of  $g_1$  and  $g_2$  with respect to  $\lambda$  for various  $\xi$ . As  $\lambda$  goes to infinity,  $g_1$  and  $g_2$  tend to be zero. Therefore, the results reduce to that of perfect interface. As  $\lambda$  equals to zero,  $g_1$  and  $g_2$  tend to  $-2$  and  $2$ , respectively, the results reduce to that for the free surface. The numerical results in Fig. 6 show that  $g_1$  and  $g_2$  have weak dependence on  $\xi$ .

### 5. Concluding remarks

The imperfect interface introduces a new variable,  $\lambda$ , into the problem of interaction between the dislocation and interface. The results on a screw dislocation near planar interface or circular inclusion show that the force acting on the dislocation due to the presence of the imperfect interface is between the values for a free surface and that for a perfectly bonded interface. The dimensionless parameter  $\lambda$  collects the effects of the imperfect interface on the screw dislocation.

The forces acting on the dislocation for all three configurations were given as functions of  $\lambda$ . The forms of the equations, Eqs. (25), (33), (43) and (48) are analytical and can be applied to other research areas, such as fracture and composite mechanics, as the fundamental solutions.

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**Appendix A. (Amazigo and Rubinfeld, 1980)**

*A.1. Definition: uniform convergence*

Suppose  $f(s, x)$  is continuous for  $s \geq s_0$  and  $x_0 \leq x \leq x_1$ . Then the improper integral  $\int_{s_0}^{\infty} f(s, x) ds$  is said to converge uniformly on  $[x_0, x_1]$  to  $F(x)$  if, for any positive number  $\epsilon$ , there is a number  $N$ , independent of  $x$ , such that, for all  $x$  in  $[x_0, x_1]$ ,

$$\left| \int_{s_0}^B f(s, x) ds - F(x) \right| < \epsilon, \quad \text{whenever } B > N. \tag{A.1}$$

*A.2. Theorem: Weierstrass M-test for uniform convergence*

If  $f(s, x)$  is continuous for  $s \geq s_0$  and  $x_0 \leq x \leq x_1$ , and if there is a function  $M(s)$  such that

$$|f(s, x)| \leq M(s), \quad s \geq s_0, \quad x_0 \leq x \leq x_1, \tag{A.2}$$

then if  $\int_{s_0}^{\infty} M(s) ds$  converges,  $\int_{s_0}^{\infty} f(s, x) ds$  converges uniformly.

*A.3. Theorem: integration of integration order*

If the function  $f(s, x)$  is continuous for  $s \geq s_0$  and  $x_0 \leq x \leq x_1$ , and the improper integral  $\int_c^{\infty} f(s, x) ds$  converges uniformly, then

$$\int_{x_0}^{x_1} \left[ \int_{s_0}^{\infty} f(s, x) ds \right] dx = \int_{s_0}^{\infty} \left[ \int_{x_0}^{x_1} f(s, x) dx \right] ds. \tag{A.3}$$

*A.4. Theorem: interchange of order of differentiate and integration*

If (i) the function  $f(s, x)$  is continuous for  $s \geq s_0$  and  $x_0 \leq x \leq x_1$ , and differentiable with respect to  $x$ , (ii) the improper integral  $\int_c^{\infty} f(s, x) ds$  converges, (iii)  $\int_c^{\infty} (\partial/\partial x)[f(s, x)] ds$  converges uniformly, then

$$\frac{\partial}{\partial x} \int_{s_0}^{\infty} f(s, x) ds = \int_{s_0}^{\infty} \frac{\partial}{\partial x} [f(s, x)] ds. \tag{A.4}$$

**Appendix B. Interchange the order of integration in elastic energy**

The elastic energy is expressed by Eq. (24),

$$\begin{aligned} E &= \frac{b}{2} \int_{a+r_0}^R \sigma_{zy}(x, 0) dx \\ &= \frac{b^2 G_1}{4\pi} \ln \left( \frac{2a}{r_0} \right) - \frac{b^2 (K-1) G_2}{4\pi} \\ &\quad \times \int_{a+r_0}^R \int_0^{\infty} \frac{\lambda e^{-s(x+a)}}{sa + \lambda} ds dx. \end{aligned} \tag{B.1}$$

The first term in Eq. (B.1) is the energy of a dislocation interacting with a free surface, which is well documented. Let us focus on the second term in Eq. (B.1), which is contributed by the interfaces

$$\frac{\lambda e^{-s(x+a)}}{as + \lambda} < e^{-(2a+r_0)s} \quad (\forall x \in [a+r_0, R])$$

and

$$\int_0^{\infty} e^{-(2a+r_0)s} ds = \frac{1}{2a+r_0}. \tag{B.2}$$

According to the Weierstrass M-test of Appendix A.2,

$$\int_0^{\infty} \frac{\lambda e^{-s(x+a)}}{as + \lambda} ds$$

is uniformly convergent with respect to  $x$  on  $[a + r_0, R]$ , therefore the order of integration can be interchanged,

$$\begin{aligned} & \int_{a+r_0}^R \int_0^\infty \frac{\lambda e^{-s(x+a)}}{as + \lambda} ds dx \\ &= \int_0^\infty \int_{a+r_0}^R \frac{\lambda e^{-s(x+a)}}{as + \lambda} dx ds \\ &= \int_0^\infty \frac{\lambda}{as + \lambda} \left( \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right) ds. \end{aligned} \quad (B.3)$$

Eq. (B.3) leads to Eq. (24).

### Appendix C. Calculation of force on dislocation

Since conditions (i) and (ii) in Appendix A.4 have been satisfied for Eq. (B.3), hereby we only consider condition (iii). It is found that

$$\begin{aligned} & \int_0^\infty \frac{\partial}{\partial a} \left[ \frac{\lambda}{as + \lambda} \left( \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right) \right] ds \\ &= - \int_0^\infty \frac{\lambda}{as + \lambda} (2e^{-(2a+r_0)s} - e^{-(a+R)s}) ds. \end{aligned} \quad (C.1)$$

It should be pointed out that  $\lambda/(as + \lambda)$  is independent of  $a$  (see the definition of  $\lambda$  in Eq. (17)). For the right-hand side of Eq. (C.1), we have

$$\frac{\lambda}{as + \lambda} (2e^{-(2a+r_0)s} - e^{-(a+R)s}) < 2e^{-r_0s} \quad \text{and} \quad (C.2)$$

$$\int_0^\infty e^{-r_0s} ds = \frac{1}{r_0}.$$

Again, according to the Weierstrass M-test of Appendix A.2,

$$\int_0^\infty \frac{\partial}{\partial a} \left[ \frac{\lambda}{as + \lambda} \left( \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right) \right] ds$$

is uniformly convergent with respect to  $a$  on  $[0, \infty)$ . Thus,

$$\begin{aligned} & \frac{\partial}{\partial a} \int_0^\infty \left[ \frac{\lambda}{as + \lambda} \left( \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right) \right] ds \\ &= \int_0^\infty \frac{\partial}{\partial a} \left[ \frac{\lambda}{as + \lambda} \left( \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right) \right] ds \\ &= -\frac{1}{a} \left[ \frac{2}{2 + r_0/a} \left( 2\lambda + \frac{r_0}{a} \lambda \right) \right. \\ & \quad \times e^{2\lambda + (r_0/a)\lambda} \text{Ei} \left( 2\lambda + \frac{r_0}{a} \lambda \right) - \frac{1}{1 + R/a} \\ & \quad \times \left( \lambda + \frac{R}{a} \lambda \right) e^{\lambda + (R/a)\lambda} \text{Ei} \left( \lambda + \frac{R}{a} \lambda \right) \left. \right]. \end{aligned} \quad (C.3)$$

When  $r_0 \rightarrow 0$  and  $R \rightarrow \infty$ , Eq. (C.3) reduces to

$$\begin{aligned} & \frac{\partial}{\partial a} \int_0^\infty \left[ \frac{\lambda}{as + \lambda} \left( \frac{e^{-(2a+r_0)s} - e^{-(a+R)s}}{s} \right) \right] ds \\ &= -\frac{1}{a} 2\lambda e^{2\lambda} \text{Ei}(2\lambda). \end{aligned} \quad (C.4)$$

Eq. (25) is obtained.

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