thermal FM which is inphase with respect to the carrier density FM.

Discussion: A small-signal analysis of the laser rate equations yields a relationship between the change in optical frequency $\Delta f_{\text{op}}$ and the change in photon density $\Delta S$ given by

$$\Delta f_{\text{op}} = \frac{1}{\gamma_p \gamma_r} \left( \omega_0 + \gamma_r \right) \Delta S$$  

(1)

where $\gamma_r$ is the photon lifetime, $\omega_0$ is the relaxation oscillation frequency, $\gamma_r$ is the modulation frequency and $\gamma_r$ is the nonlinear damping factor. Because of the nonlinear damping term in eqn. 1, the FM-AM phase difference is zero at low frequencies, and increases to $90^\circ$ when $\omega_0 > \gamma_r$. Furthermore, since $\gamma_r$ increases linearly with optical power, the phase difference should increase with frequency at a slower rate as the output power increases. These predictions are borne out by the phase difference data shown in the lower portion of Fig. 3. Also, at a given optical power, the value of $\gamma_r$ can be determined from the measurement of the FM-AM phase difference with frequency. Similarly, eqn. 1 accounts for the enhancement of the magnitude of the FM response relative to the AM response, as reported above, resulting in an FM bandwidth which is larger than the AM bandwidth.

Conclusion: The FM and AM responses of a 1530 nm DFB laser in a high-speed driver circuit have been measured for modulation frequencies up to 15 GHz using a novel birefringent fibre interferometer. At frequencies above a few GHz, the FM response was enhanced relative to the AM response. A 12 GHz FM bandwidth was obtained for output powers between 5.5 and 9.0 mW. The phase difference between the FM and AM responses increased from 0 to 90° as the modulation frequency increased. At high optical power levels, the phase difference increased nearly linearly with frequency, with a slope corresponding to a 15-25 ps delay between the FM and AM responses. The data are well explained by the laser rate equations and can be used to determine the nonlinear damping factor $\gamma_r$. These results show promise for FSK modulation of DFB lasers at bit rates in the range 10-20 Gbit/s.

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R. S. VODHANEL

Bellcore
331 Newman Springs Road
Red Bank, NJ 07701-7600, USA

S. TSUJI
Central Research Laboratory
Hitachi Ltd.
Kokubunji, Tokyo 185, Japan

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DESIGN OF HIGHLY SELECTIVE TWO-DIMENSIONAL RECURSIVE FAN FILTERS BY RELAXING SYMMETRY CONSTRAINTS

Indexing terms: Filters, Recursive filters, Transfer functions

We show for the design of quadratically symmetric 2-D fan filters that it is unnecessarily restrictive to prescribe exact quadrantal symmetry, which requires that the denominator of the Z-transform transfer function be product-separable. Superior approximately symmetric fan filter designs can be achieved using nonseparable denominators.

Introduction: A wide variety of useful two-dimensional (2-D) discrete filter transfer functions $H(z_1, z_2)$ possess quadrantal symmetry in their magnitude frequency response; that is, 

$$|H(e^{j\omega_1}, e^{j\omega_2})| = |H(e^{-j\omega_1}, e^{j\omega_2})| = |H(e^{j\omega_1}, e^{-j\omega_2})| = |H(e^{-j\omega_1}, e^{-j\omega_2})|$$

(1)

where $H(z_1, z_2)$ and $H(z_1, z_2)$ are relatively prime. It is now well established1-3 that, for such stable quadrantal symmetric $H(z_1, z_2)$, the denominator $D(z_1, z_2)$ must be product-separable; hence, it must be expressible in the form $D(z_1, z_2) = D(z_1)D(z_2)$, where $D(z_1)$ and $D(z_2)$ are strictly Hurwitz polynomials.

In this letter we investigate the design of the widely used class of quadrantal symmetric functions known as fan, or velocity, filters, as shown in Fig. 1. In designing fan filters of this form, we show that the separability constraint can be an unnecessary restriction. We show that, by relaxing the requirement of exact quadrantal symmetry in regions of the 2-D frequency plane where symmetry is unimportant, transfer functions with nonsparable denominators can be used to achieve fan filter designs that are superior to those available with separable denominators.

Fig. 1 Fan filter specification

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Relaxation of quadrantal symmetry: One of the principal benefits of imposing exact quadrantal symmetry on a 2-D function is that stability of the resulting algorithm is guaranteed by ensuring that $D(x_0)$ and $D(z_0)$ are strictly Hurwitz. However, the full advantage of employing recursion via the term $1/D(z_1, z_2)$ is only obtained if this term plays a dominant role in shaping the overall 2-D magnitude frequency response via the corresponding leading contributing term $1/[D(e^{j\omega}, e^{j\phi})]$. Enforcing quadrantal symmetry and therefore separability leads to a contributing term $1/D(e^{j\omega}D(e^{j\phi}))$ which, as a product of 1-D functions, does not lend itself to the required fan shape. Consequently, the primary benefit of forming the fan shape falls on the (nonseparable) numerator $|N(e^{j\omega}, e^{j\phi})|$

By relaxing the quadrantal symmetry constraint in regions of the 2-D frequency plane where symmetry is not important, such as the transition region, it is possible to achieve significant improvements in design. Consider the specification shown in Fig. 1, where the width of the transition region is defined by the parameters $x_1$ and $x_2$. By relaxing quadrantal symmetry in this transition region, a nonseparable denominator $D(z_1, z_2)$ may be used. We propose to approximate the passband and stopband regions as closely as possible while tolerating 'acceptable' behaviour in the transition regions. We define 'acceptable' behaviour in this case as $0 \leq |H(e^{j\omega}, e^{j\phi})| \leq 1$.

Design examples: The relaxation of quadrantal symmetry in the transition regions is a particularly useful technique for the design of low-order highly selective 2-D fan filters. We choose to design a fan-stop filter having a total angular stopband width $\theta = 20^\circ$. The order of the filter is $(M, N) = (5, 2)$ and the design is carried out using an existing 2-D recursive filter design program, specifically modified to conform to the relaxed specification in Fig. 1. For $x_1 = 0.0525\pi$ and $x_2 = 0.18\pi$. An expanded view of the stopband of $|H(e^{j\omega}, e^{j\phi})|$ for the completed design is shown in Fig. 2, and the coefficients $N = [a_{nm}]$ and $D = [b_{nm}]$ in Eqn 1 are given by

\[N = \begin{bmatrix}
-0.348263 & 0.672017 & -0.2576901 \\
0.902289 & -1.998527 & 0.8605637 \\
-0.292910 & 1.031212 & -0.5784674 \\
1.157559 & 2.165731 & -0.8327808 \\
1.332481 & -2.828259 & 1.260246 \\
-0.4363195 & 0.9466001 & -0.4519792 \\
\end{bmatrix}
\]

\[D = \begin{bmatrix}
1.0 & -1.198685 & 0.3595865 \\
-2.188287 & 2.820891 & -0.9020295 \\
0.934936 & -1.519597 & 0.5785126 \\
1.78034 & -1.257305 & 0.3193958 \\
-1.234083 & 1.925151 & -0.5103743 \\
0.3274713 & -0.4602485 & 0.1620259 \\
\end{bmatrix}
\]

Conclusion: We have shown that for the requirement for exact quadrantal symmetry in the design of 2-D quadrantly symmetric fan filters can impose an unnecessary restriction. By relaxing quadrantal symmetry constraints in the transition regions, it is possible to employ filters having nonseparable denominators and near-quadrantal symmetry. We have shown that, for fan filters having a given low order (e.g. 5, 2), the nonseparable design is significantly superior to the separable design.

Z. LIN
L. T. BRUTON
N. R. BARTLEY
Department of Electrical Engineering
University of Calgary
2500 University Drive NW
Calgary, Alberta, Canada T2N 1N4

References

**EFFICIENT OPERATION OF ARRAY-PUMPED Er**\(^{3+}\) **DOPED SILICA FIBRE LASER AT 1-6µm**

**Indexing terms:** Optical fibres, Lasers and laser applications

We present the results for diode-array pumped Er\(^{3+}\) doped silica fibres, which lase near 1550 nm. A maximum output power of 8 mW is obtained with a 13% overall efficiency against the launched pump power.

**Introduction:** Diode laser pumped operation of the 1.5 µm transition in Er\(^{3+}\) doped silica fibres has been reported recently.\(^{1,2}\) This transition is of particular interest because of the