Optimizing Lifetime for Continuous Data Aggregation with Precision Guarantees in Wireless Sensor Networks

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Abstract—This paper exploits the tradeoff between data quality and energy consumption to extend the lifetime of wireless sensor networks. To obtain an aggregate form of sensor data with precision guarantees, the precision constraint is partitioned and allocated to individual sensor nodes in a coordinated fashion. Our key idea is to differentiate the precisions of data collected from different sensor nodes to balance their energy consumption. Three factors affecting the lifetime of sensor nodes are identified: (1) the changing pattern of sensor readings; (2) the residual energy of sensor nodes; and (3) the communication cost between the sensor nodes and the base station. We analyze the optimal precision allocation in terms of network lifetime and propose an adaptive scheme that dynamically adjusts the precision constraints at the sensor nodes. The adaptive scheme also takes into consideration the topological relations among sensor nodes and the effect of in-network aggregation. Experimental results using real data traces show that the proposed scheme significantly improves network lifetime compared to existing methods.

Index Terms—data aggregation, data accuracy, energy efficiency, network lifetime, sensor network.

I. INTRODUCTION

WIRELESS sensor networks are used in a wide range of applications to capture, gather and analyze live environmental data [1], [2]. A wireless sensor network typically consists of a base station and a group of sensor nodes (see Figure 1). The sensor nodes are responsible for continuously sampling physical phenomena such as temperature and humidity. They are also capable of communicating with each other and the base station through radios. The base station, on the other hand, serves as a gateway for the sensor network to exchange data with applications to accomplish their missions.

While the base station can have continuous power supply, the sensor nodes are usually battery-powered. The batteries are inconvenient and sometimes even impossible to replace. When a sensor node runs out of energy, its coverage is lost. The mission of a sensor application would not be able to continue if the coverage loss is remarkable. Therefore, the practical value of a sensor network is determined by the time duration before it fails to carry out the mission due to insufficient number of “alive” sensor nodes. This duration is referred to as the network lifetime [1]. It is both mission-critical and economically desirable to manage sensor data in an energy-efficient way to extend the lifetime of sensor networks.

The data captured by the sensor nodes are often converted into an aggregate form requested by the applications (e.g., average temperature reading). Primarily designed for monitoring purposes, many sensor applications require continuous aggregation of sensor data [3]. Exact data aggregation requires substantial energy consumption because each sensor node has to report every reading to the base station. In wireless sensor networks, communication is a dominant source of energy consumption [4], [5]. To save energy, data semantics can be relaxed to allow approximate data aggregation with precision guarantees [6], [7], [8], [9]. The precision can, for example, be specified in the form of quantitative error bounds: “average temperature reading of all sensor nodes within an error bound of 1°C.” In this way, the sensor nodes do not have to report all readings to the base station. Only the updates necessary to guarantee the desired level of precision need to be sent.

It is, however, a challenging task to optimize network lifetime under approximate data aggregation because the sensor nodes are inherently heterogeneous in energy consumption. First, when the data captured by different sensor nodes change at different magnitudes and frequencies, the sensor nodes may report data at different rates. Second, the wireless communication cost depends on the transmission distance [10], [11]. Due to the geographically distributed nature of sensor networks, the sensor nodes are likely to differ significantly in the energy cost of sending a message to the base station. Even if all sensor nodes report data at the same rate, their energy consumption can be highly unbalanced, thereby reducing network lifetime. In addition to reporting local sensor readings, the intermediate nodes in a multi-hop network are also responsible for relaying...
the data originated from other nodes to the base station. The nodes closer to the base station normally relay larger amounts of data than the nodes farther away from the base station.

In this paper, we investigate the optimization of network lifetime for approximate data aggregation. We leverage the semantics of approximate data aggregation in balancing the energy consumption of the sensor nodes. Our key idea is to differentiate the quality of data collected from different sensor nodes by partitioning the precision constraint of data aggregation among the sensor nodes in a coordinated fashion. Our contributions are summarized as follows:

- We identify three factors affecting the lifetime of sensor nodes in the context of approximate data aggregation: (1) the changing pattern of sensor readings; (2) the residual energy of the sensor nodes; and (3) the communication cost between the sensor nodes and the base station. We then analyze the optimal precision allocation in terms of network lifetime.
- We develop a candidate-based method for precision allocation and prove its optimality for single-hop networks. Based on this method, an adaptive scheme is proposed to dynamically adjust the error bounds allocated to the sensor nodes. The adjustment period is also dynamically set to control the communication overhead.
- We derive the hardness results of candidate-based precision allocation in multi-hop networks. We extend the adaptive scheme to work in multi-hop networks by taking into consideration the effect of in-network aggregation and the topological relations among the sensor nodes.
- We present an experimental evaluation using real data traces over a wide range of system configurations. The results show that the proposed scheme significantly improves network lifetime compared to existing methods.

The rest of this paper is organized as follows. Section II summarizes the related work. Section III describes the system model and gives some basic definitions. Section IV analyzes the optimal precision allocation in single-hop networks and then proposes an adaptive precision allocation scheme. Section V extends the adaptive scheme to multi-hop networks. The experimental setup and results are discussed in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORK

Wireless sensor networks have attracted much research effort in recent years. From the networking perspective, researchers have primarily focused on optimizing network related operations such as routing and media access [12], [13], [14], [15]. From the database perspective, researchers have mainly focused on query processing over sensor data [16], [17], [18], [19]. However, not much work has looked into trading data quality for energy efficiency.

Recently, several approaches have been proposed to relax data semantics and allow a specified degree of inaccuracy to be tolerated in sensor data collection. To acquire approximate readings of individual sensor nodes, the precision constraints can be set independently for different sensor nodes [8], [20]. In contrast, to collect an aggregate form of sensor data over the network, the precision settings of different sensor nodes should be inter-related. Olston et al. [6] investigated burden-based precision adjustment for continuous queries over distributed data streams. However, they did not model in-network aggregation, which is a commonly used technique to reduce the traffic of data collection in wireless sensor networks [21]. Sharaf et al. [7] implemented a simple uniform precision allocation for in-network sensor data aggregation. Deligiannakis et al. [9] further optimized the allocation to reduce the number of messages transmitted in the network. However, none of these studies has taken energy and lifetime models into consideration. Thus, their proposed techniques are not effective in handling the energy constraints in wireless sensor networks. As shall be shown by our experimental results, minimizing the total network traffic does not necessarily optimize network lifetime. Different from existing work, in this paper, we aim at extending network lifetime for data aggregation with precision guarantees in sensor networks.

Considine et al. [22] and Nath et al. [23] implemented approximate data aggregation in the presence of multi-path routing by means of sketches and synopses. However, they did not make use of temporal locality to suppress data updates. Deshpande and Chu et al. [24], [25] applied statistical techniques to model the distributions of sensor data for approximate data collection. The performance of this approach depends on the quality of the models built. Different from this approach, we do not require the construction of statistical models in advance. Our proposed techniques dynamically adapt to the changing pattern of sensor readings on the fly. Related work on approximate data collection also includes representing sensor readings with sophisticated data structures [26], [27] and exploiting the spatial correlation between sensor readings [28], [29]. These studies are complementary to our work.

III. PRELIMINARIES

We consider data aggregation with precision guarantees in a network of $n$ sensor nodes. The sensor nodes are geographically distributed in an operational area. They periodically sample the local phenomena such as temperature and humidity. Without loss of generality, the sampling period is assumed to be 1 time unit. The base station collects data from the sensor nodes and feeds them to an application. The application specifies the precision constraint of data aggregation by an upperbound $E$ (called the error bound) on the quantitative difference between an approximate result and the exact result [7], [9]. That is, on receiving an aggregate result $A'$ from the sensor network, the application would like to be assured that the exact aggregate result $A$ lies in the interval $[A' - E, A' + E]$.

In approximate data aggregation, not all sensor readings have to be sent to the base station. To reduce communication cost, the designated error bound on aggregate data can be partitioned and allocated to individual sensor nodes (we shall call it precision allocation). Each sensor node updates a new reading with the base station only when the new reading significantly deviates from the last update to the base station and violates the allocated error bound. To guarantee the designated precision of aggregate data, the error bounds allocated
to individual sensor nodes have to satisfy certain feasibility constraints. Different aggregation functions impose different constraints. In this paper, we consider three commonly used types of aggregations: SUM, COUNT and AVERAGE. For SUM and COUNT aggregations, to guarantee an error bound \( E \) on aggregate data, the total error bound allocated to the sensor nodes cannot exceed \( E \), i.e.,

\[
\sum_{i=1}^{n} e_i \leq E, \quad (1)
\]

where \( e_i \) is the error bound allocated to node \( i \). For AVERAGE aggregation, the total error bound allocated to the sensor nodes cannot exceed \( n \cdot E \), i.e.,

\[
\sum_{i=1}^{n} e_i \leq n \cdot E, \quad (2)
\]

where \( n \) is the number of sensor nodes.

Eligible precision allocation under the feasibility constraint is not unique. For example, in a network of 10 temperature sensor nodes, if the given error bound on AVERAGE aggregation is 1°C, we can allocate an error bound of 1°C to each sensor node. Alternatively, we can also allocate an error bound of 5.5°C to a selected node and an error bound 0.5°C to each of the remaining nodes. This offers the flexibility to adjust the energy consumption of individual sensor nodes by careful precision allocation. In general, to collect the readings of a sensor node at higher precision (i.e., smaller error bound), the sensor node needs to send data updates to the base station more frequently, which introduces higher energy consumption.

We denote the energy consumed by sensor node \( i \) to send and receive a data update by \( s_i \) and \( v_i \) respectively. They can take different forms to cater for a wide range of factors. In the simplest case, if all sensor nodes use a default radio communication range, \( s_i \)'s are the same for all nodes. More sophisticatedly, if the sensor nodes know the locations of the receivers [30], [31], [11], they can adapt the power level to the transmission distance. The sensor nodes with longer transmission distances would be associated with higher \( s_i \)'s. In addition, reliability can also be modeled in the energy cost. The sensor nodes incident to less reliable links are entitled to higher \( s_i \)'s and \( v_i \)'s due to possible retransmissions. The exact forms of \( s_i \) and \( v_i \) are orthogonal to our analysis and beyond the scope of this paper. We simply assume that each sensor node \( i \) knows \( s_i \) and \( v_i \).

Similar to other studies [32], [33], [34], [35], we define the network lifetime as the time duration before the first sensor node runs out of energy. Our analysis is also applicable to redundant sensor deployment where each location of interest is covered by several sensor nodes. From the viewpoint of network lifetime, the set of sensor nodes monitoring the same location can be converted to an equivalent single node by adding up the energy budgets of these sensor nodes. More generally, if the network lifetime is defined as the time duration before a given portion of sensor nodes run out of energy, our proposed scheme can be applied repeatedly after the exhaustion of a sensor node’s energy.

IV. Precision Allocation in Single-Hop Networks

We start by investigating the precision allocation in a single-hop network where each sensor node sends its local readings to the base station directly. Single-hop networks are preferred in some situations due to a number of reasons [11]. Moreover, the analysis of precision allocation in a single-hop network also provides insights on the allocation in a multi-hop network. The adaptive precision allocation scheme developed for single-hop networks will serve as a building block of the scheme we shall propose for multi-hop networks in Section V.

Note that constraints (1) and (2) share the characteristic that the total error bound of the sensor nodes is capped by a given value. We shall focus on constraint (1) in our discussion. The analysis and algorithms developed in this paper can be adapted to handle constraint (2) in a straightforward manner. They are also directly applicable to SUM and AVERAGE aggregations over any fixed subset of the sensor nodes.

A. Analysis of Optimal Precision Allocation

Consider a snapshot of the network. Let \( e_1, e_2, \ldots, e_n \geq 0 \) be the error bounds currently allocated to sensor nodes 1, 2, \ldots, \( n \) respectively. The quantitative relationship between the rate of data updates sent by a sensor node and its allocated error bound depends on the changing pattern of sensor readings. Without loss of generality, we shall denote the update rate of each sensor node \( i \) as a function \( u_i(e_i) \) of the allocated error bound \( e_i \geq 0 \). \( u_i(e_i) \) is essentially the rate at which node \( i \)'s reading changes by more than \( e_i \). Intuitively, \( u_i(e_i) \) is a non-increasing function with respect to \( e_i \), and \( u_i(\infty) = 0 \).

Since the sensor nodes in a single-hop network are not involved in relaying data from other sensor nodes to the base station, the energy consumption rate of node \( i \) is simply

\[
u_i(e_i) \cdot s_i,
\]

where \( s_i \) refers to the energy cost for node \( i \) to send a data update to the base station. Suppose the residual energy of node \( i \) is \( p_i \). Then, the expected lifetime of node \( i \) is

\[
\frac{p_i}{u_i(e_i) \cdot s_i}.
\]

Therefore, the network lifetime is given by

\[
\min_{1 \leq i \leq n} \frac{p_i}{u_i(e_i) \cdot s_i}.
\]

The objective of precision allocation is to find a set of error bounds \( e_1, e_2, \ldots, e_n \) that maximize the network lifetime under the constraint

\[
\sum_{i=1}^{n} e_i \leq E.
\]

We now analyze the optimal precision allocation. For simplicity, we shall assume functions \( u_i(\cdot) \)'s are continuous and denote the inverse function of \( u_i(\cdot) \) by \( u_i^{-1}(\cdot) \).

Since \( u_i(\cdot) \) is non-increasing, the minimum lifetime of sensor node \( i \) is given by

\[
l_i = \frac{p_i}{u_i(0) \cdot s_i}.
\]
Without loss of generality, suppose
\[ l_1 \leq l_2 \leq \cdots \leq l_n. \]

For each pair of nodes \( i \) and \( j \) where \( i \leq j \), consider the error bound \( u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) \) that makes the lifetime of node \( i \) equivalent to the minimum lifetime of node \( j \). Since \( u_i(\cdot) \) is non-increasing, it follows from \( l_j \leq l_{j+1} \) that
\[ u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) \leq u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}). \]
Thus, given any \( 1 \leq j < n \),
\[ \sum_{i=1}^{j} u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) \leq \sum_{i=1}^{j} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}) = u_1^{-1}(u_j(0)) + \sum_{i=1}^{j} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}) \]
\[ = u_1^{-1}(u_j(0)) + \sum_{i=1}^{j} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}) \]
\[ = \sum_{i=1}^{j+1} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}). \]
This implies \( \sum_{i=1}^{j} u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) \) is non-decreasing with increasing \( j \). Note that when \( j = 1 \),
\[ \sum_{i=1}^{j} u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) = u_1^{-1}(u_1(0)) = 0. \]

Therefore, given an error bound \( E > 0 \) on data aggregation, if \( \sum_{i=1}^{n} u_i^{-1}(\frac{p_i}{l_n \cdot s_i}) > E \), there must exist a \( j^* \), where \( 1 \leq j^* < n \), such that
\[ \sum_{i=1}^{j^*} u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) \leq E < \sum_{i=1}^{j^*+1} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}). \]
Since
\[ \sum_{i=1}^{j^*+1} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}) = \sum_{i=1}^{j^*} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}), \]
we have
\[ \sum_{i=1}^{j^*} u_i^{-1}(\frac{p_i}{l_j \cdot s_i}) \leq E < \sum_{i=1}^{j^*} u_i^{-1}(\frac{p_i}{l_{j+1} \cdot s_i}). \]

Hence, there also exists an \( l^* \), where \( l_j \leq l^* < l_{j+1} \), such that
\[ \sum_{i=1}^{j^*} u_i^{-1}(\frac{p_i}{l^* \cdot s_i}) = E. \]
(3)

On the other hand, if \( \sum_{i=1}^{n} u_i^{-1}(\frac{p_i}{l_n \cdot s_i}) \leq E \), since \( u_i(\cdot) \)’s are non-increasing and \( u_i^{-1}(0) = \infty \), there exists an \( l^* \), where \( l^* \geq l_n \), such that
\[ \sum_{i=1}^{n} u_i^{-1}(\frac{p_i}{l^* \cdot s_i}) = E. \]
(4)

For convenience, we shall denote \( j^* = n \) in this case so that (4) is consistent with (3).

**Theorem 1:** An optimal precision allocation is given by
\[ e_i^* = \begin{cases} u_1^{-1}(\frac{p_i}{l^* \cdot s_i}) & 1 \leq i \leq j^*, \\ 0 & j^* < i \leq n. \end{cases} \]
This allocation has a lifetime \( l^* \).

**Proof:** It follows from (3) that \((e_1^*, e_2^*, \ldots, e_n^*)\) satisfies the feasibility constraint of precision allocation. Assume on the contrary that there exists another precision allocation \((e_1', e_2', \ldots, e_n')\) which has a lifetime \( l' > l^* \). The definition of network lifetime implies that for any \( 1 \leq i \leq j^* \),
\[ \frac{p_i}{u_i(e_i') \cdot c_i} \geq \frac{p_i}{u_i(e_i^*) \cdot c_i}. \]
Thus,
\[ u_i(e_i') > u_i(e_i^*). \]
Since \( u_i(\cdot) \) is non-increasing, we have \( e_i' > e_i^* \).
Therefore,
\[ \sum_{i=1}^{n} e_i' \geq \sum_{i=1}^{j^*} e_i > \sum_{i=1}^{j^*} e_i^* = E, \]
which contradicts the feasibility of \((e_1', e_2', \ldots, e_n')\).

Hence, the theorem is proven.

**B. Candidate-Based Precision Allocation**

In practice, the exact forms of \( u_i(\cdot) \)’s (i.e., the changing patterns of sensor readings) may not be known a priori and they may even change dynamically. Thus, we propose a candidate-based method for precision allocation. The key idea is to let each sensor node estimate and report to the base station the normalized energy consumption rates for a number of candidate error bounds based on historical sensor readings. The base station optimizes precision allocation based on these candidates to extend network lifetime. Since the general relationships between error bounds and update rates are not known, we restrict the error bound of each sensor node to one of its candidates. Such allocations are called candidate precision allocations and the one that maximizes network lifetime is called the optimal candidate precision allocation.

Assume that each sensor node chooses \( m \) candidates. For each node \( i \), let \( e_{i,1} < e_{i,2} < \cdots < e_{i,m} \) be the list of candidate error bounds, and \( r_{i,1}, r_{i,2}, \cdots, r_{i,m} \) be the corresponding normalized energy consumption rates. It follows that
$r_{i,1} \geq r_{i,2} \geq \cdots \geq r_{i,m}$. Suppose the smallest candidate error bounds for the sensor nodes do not add up to the designated bound on data aggregation, i.e., $e_{1,1} + e_{2,1} + \cdots + e_{n,1} \leq E$. Algorithm 1 presents the pseudocode to compute the optimal candidate precision allocation.

**Algorithm 1** Optimal Candidate Precision Allocation in a Single-Hop Network

**Input:**
- $E$: error bound of data aggregation
- $e_{i,x}$: candidate error bounds and normalized energy consumption rates

**Output:**
- $e_{i,x}$: error bound of each node in optimal allocation

```plaintext
1: for $i = 1$ to $n$ do
2:     $x_i = 1$;
3: end for
4: while $\min_{1 \leq i \leq n} x_i \neq m$ do
5:     $j = \arg \max_{1 \leq i \leq n, x_i \neq m} r_{i,x_i}$;
6:     if $e_{j,x_j+1} + \sum_{i \neq j} e_{i,x_i} > E$ then
7:         break;
8:     end if
9:     $x_j = x_j + 1$;
10: end while
```

Initially, the error bound of each sensor node is set to its smallest candidate (steps 1 to 3). In each iteration of steps 4 to 10, the error bound of the node having the highest energy consumption rate is replaced with its next smallest candidate. The iteration stops if a new replacement would make the total error bound of the sensor nodes exceed the designated bound on data aggregation (steps 6 to 7). The worst-case time complexity of Algorithm 1 is $O(mn)$. We show that Algorithm 1 produces an optimal candidate precision allocation.

**Theorem 2:** The candidate precision allocation computed by Algorithm 1 maximizes network lifetime.

**Proof:** Let $(e_{1,x_1}, e_{2,x_2}, \ldots, e_{n,x_n})$ be the precision allocation computed by Algorithm 1. It is obvious that $(e_{1,x_1}, e_{2,x_2}, \ldots, e_{n,x_n})$ is feasible. Suppose under such allocation, sensor node $k$ has the highest normalized energy consumption rate, i.e.,

$$r_{k,x_k} = \max_{1 \leq i \leq n} r_{i,x_i}.$$  

The network lifetime is then given by

$$\frac{1}{r_{k,x_k}} = \max_{1 \leq i \leq n} \frac{1}{r_{i,x_i}}.$$  

It is easy to infer from Algorithm 1 that: (i) if $x_k < m$,

$$e_{k,x_k+1} + \sum_{i \neq k} e_{i,x_i} > E;$$  

and (ii) for each $x_i > 1$ where $i \neq k$,

$$r_{i,x_i-1} \geq r_{k,x_k}.$$  

Assuming on the contrary that there exists another candidate precision allocation $(e_{1,x_1}', e_{2,x_2}', \ldots, e_{n,x_n}')$ with a longer network lifetime, i.e.,

$$\frac{1}{\max_{1 \leq i \leq n} r_{i,x_i}} > \frac{1}{\max_{1 \leq i \leq n} r_{i,x_i}}.$$  

It follows that

$$\max_{1 \leq i \leq n} r_{i,x_i} < \max_{1 \leq i \leq n} r_{i,x_i}.$$  

Since

$$r_{k,x_k} = \max_{1 \leq i \leq n} r_{i,x_i},$$

we have

$$r_{k,x_k}' \leq \max_{1 \leq i \leq n} r_{i,x_i}' < r_{k,x_k}.$$  

Therefore,

$$x_k < x_k' \leq m,$$

and hence,

$$e_{k,x_k+1} \leq e_{k,x_k'}.$$  

Based on property (i),

$$e_{k,x_k} + \sum_{i \neq k} e_{i,x_i} \geq e_{k,x_k+1} + \sum_{i \neq k} e_{i,x_i} > E \geq \sum_{i=1}^{n} e_{i,x_i} = e_{k,x_k'} + \sum_{i \neq k} e_{i,x_i}'.$$

Thus, there must exist a $j \neq k$ such that

$$e_{j,x_j} > e_{j,x_j'},$$

which implies

$$x_j - 1 \geq x_j' \geq 1.$$  

It follows from property (ii) that

$$r_{j,x_j} \geq r_{j,x_j-1} \geq r_{k,x_k}.$$  

Therefore,

$$\max_{1 \leq i \leq n} r_{i,x_i} \geq r_{j,x_j} \geq r_{k,x_k} = \max_{1 \leq i \leq n} r_{i,x_i},$$

which contradicts the assumption that $(e_{1,x_1}', e_{2,x_2}', \ldots, e_{n,x_n}')$ has a longer network lifetime.

Hence, the theorem is proven.  

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1Our proposed candidate selection method (to be discussed later in this section) satisfies this constraint.

2As shall be shown by our experimental results (Section VI), a small $m$ like 5 is sufficient to achieve near optimal network lifetime.
C. Adaptive Precision Allocation

We now present an adaptive precision allocation scheme that works by adjusting the error bounds of the sensor nodes periodically. The interval between two successive adjustments is called an adjustment period. At the beginning of an adjustment period, each sensor node $i$ selects a list of candidate error bounds $e_{i,1}, e_{i,2}, \ldots, e_{i,m}$. The node keeps track of the update counts under these error bounds as it captures new readings. At the end of the adjustment period, node $i$ normalizes the counts by the length of period to obtain the data update rate $u_{i,j}$ for each $e_{i,j}$. Node $i$ then computes the normalized energy consumption rate $r_{i,j}$ for each $e_{i,j}$ by

$$r_{i,j} = \frac{u_{i,j} \cdot s_i}{p_i},$$

where $p_i$ is the present residual energy of node $i$. Node $i$ sends a candidate report message including the $e_{i,j}$’s and $r_{i,j}$’s to the base station. On receiving the messages from all sensor nodes, the base station computes the optimal precision allocation $(e_{1,1}, e_{2,1}, \ldots, e_{n,m})$ using Algorithm 1. In case $\sum_{i=1}^{n} e_{i,x_i} < E$, the leftover error bound $E - \sum_{i=1}^{n} e_{i,x_i}$ is simply allocated to the node with the highest normalized energy consumption rate since doing so would only extend network lifetime. Finally, the base station sends a precision allocation message to the sensor nodes including the new error bounds for their adjustments.

Algorithm 1 and Theorem 2 are generic in that they are applicable to any list of candidates. In this paper, we propose to choose a set of candidate error bounds that are exponentially spaced. The closer the candidates to the current error bound, the smaller the difference between neighboring candidates. The motivation is to adjust the error bounds at coarse granularity when they are far away from the optimum, and adjust them at fine granularity when they are close to the optimum. Let $e_i$ be the current error bound of sensor node $i$. Then, the candidate error bounds of node $i$ range from $\frac{1}{2} e_i$ to $\frac{3}{2} e_i$. Given the number of candidates $m = 2k + 1$, the candidate error bounds are selected as

$$\frac{1}{2} e_i, \frac{3}{4} e_i, \ldots, \frac{2k - 1}{2k - e_i}, e_i, \frac{2k + 1}{2k - e_i}, \ldots, \frac{5}{4} e_i, \frac{3}{2} e_i.$$

Note that the network lifetime is determined by the lifetime of the most energy-consuming node. Thus, to control the energy overhead of adjustments, we propose to cap the energy overhead at the most energy-consuming node by a given portion $\alpha$ of its energy budget. This is done by dynamically adapting the adjustment period at each adjustment. Specifically, each sensor node $i$ counts the number of data updates sent to the base station in the adjustment periods. At an adjustment, node $i$ estimates its energy consumption rate by $N^i s_i / L^i$, where $N^i$ is the update count in the past adjustment period, $s_i$ is the energy cost for sending, and $L^i$ is the duration of the past adjustment period. Note that at an adjustment, each sensor node needs to send a candidate report message to and receive a precision allocation message from the base

station. Thus, the energy cost at node $i$ due to an adjustment is $s_i + v_i$, where $s_i$ and $v_i$ are the sending and receiving costs respectively. To limit it at a portion $\alpha$ of the energy consumed by node $i$, the duration of the next adjustment period $L_i$ should be set such that

$$\frac{s_i + v_i}{L_i} = \alpha \cdot \frac{N^i s_i}{L^i},$$

i.e.,

$$L_i = \frac{L^i (s_i + v_i)}{\alpha N^i s_i}.$$

Each sensor node $i$ computes $L_i$ and includes it in the candidate report message sent to the base station at the end of an adjustment period. Among all $L_i$’s received, the base station selects the lowest one $L$ as the next adjustment period so as to cap the adjustment overhead at a portion $\alpha$ of the energy consumed at the most consuming node. $L$ is then included in the precision allocation message sent by the base station to all sensor nodes. We shall investigate the impact of $\alpha$ with simulation experiments in Section VI.

V. Precision Allocation in Multi-Hop Networks

A. Modeling In-Network Aggregation

If the base station is beyond the radio coverage of some sensor nodes, a multi-hop routing infrastructure has to be set up to transport data from the sensor nodes to the base station. A common practice is to organize the sensor nodes into a tree structure rooted at the base station [21]. In-network aggregation is often used to reduce the network traffic of data collection in multi-hop networks [21], [7], [9], [27]. In this approach, each intermediate node aggregates the data received from its children before forwarding them upstream in order to cut down the volume of data sent over the upper-level links in the tree. As a result, the data sent by an intermediate node to its parent is a partial aggregate result of the sensor readings in the subtree rooted at the intermediate node.

Like that in a single-hop network, each sensor node $i$ is allocated an error bound $e_i$ to control its reporting of data updates to the parent. We shall call it node $i$’s local error bound. The operation of a leaf node in a multi-hop network is the same as that in a single-hop network: it updates the parent node with a new reading whenever the new reading differs from the last reported reading by more than the local error bound. For each intermediate node, the local error bound is applied to the partial aggregate results at the node rather than its local readings [9]. To do so, each intermediate node maintains the latest data value reported by each child. At each sampling period, the intermediate node re-aggregates these data values together with its new local reading. It sends the new partial aggregate result to the parent only when the result has changed beyond its local error bound since the last update to the parent. For SUM aggregation, the partial aggregate result is the sum of the sensor readings in the subtree rooted at the intermediate node. In this way, the aggregate result collected by the base station is guaranteed to be within an error bound $\sum_{i=1}^{n} e_i$ from the exact aggregate result over the network [9].

In fact, it can be shown by induction that for each node $i$, the data value maintained by $i$’s parent node for $i$ differs from
the exact aggregate result over the subtree $T_i$ rooted at node $i$ by at most $\sum_{j \in T_i} e_j$. The correctness of this claim is trivial for any leaf node. Suppose it is also true for any child of an intermediate node $i$. Let $C_i$ be the set of $i$’s children. Then, for any node $c \in C_i$, we have

$$\left| D_{i,c} - \sum_{j \in T_c} d_j \right| \leq \sum_{j \in T_c} e_j,$$

where $d_j$ is the sensor reading at node $j$, $D_{i,c}$ is the data value maintained by node $i$ for node $c$, and $T_i$ is the subtree rooted at node $c$. Denote $i$’s parent node by $p$. According to the operation of an intermediate node presented above, we also have

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by node $p$ for node $i$. Therefore,

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by the base station for each of its child $i$. Let $C_i$ be the set of $i$’s children. Then, for any node $c \in C_i$, we have

$$\left| D_{i,c} - \sum_{j \in T_c} d_j \right| \leq \sum_{j \in T_c} e_j,$$

where $d_j$ is the sensor reading at node $j$, $D_{i,c}$ is the data value maintained by node $i$ for node $c$, and $T_i$ is the subtree rooted at node $c$. Denote $i$’s parent node by $p$. According to the operation of an intermediate node presented above, we also have

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by node $p$ for node $i$. Therefore,

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by the base station for each of its child $i$. Let $C_i$ be the set of $i$’s children. Then, for any node $c \in C_i$, we have

$$\left| D_{i,c} - \sum_{j \in T_c} d_j \right| \leq \sum_{j \in T_c} e_j,$$

where $d_j$ is the sensor reading at node $j$, $D_{i,c}$ is the data value maintained by node $i$ for node $c$, and $T_i$ is the subtree rooted at node $c$. Denote $i$’s parent node by $p$. According to the operation of an intermediate node presented above, we also have

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by node $p$ for node $i$. Therefore,

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by the base station for each of its child $i$. Let $C_i$ be the set of $i$’s children. Then, for any node $c \in C_i$, we have

$$\left| D_{i,c} - \sum_{j \in T_c} d_j \right| \leq \sum_{j \in T_c} e_j,$$

where $d_j$ is the sensor reading at node $j$, $D_{i,c}$ is the data value maintained by node $i$ for node $c$, and $T_i$ is the subtree rooted at node $c$. Denote $i$’s parent node by $p$. According to the operation of an intermediate node presented above, we also have

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by node $p$ for node $i$. Therefore,

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by the base station for each of its child $i$. Let $C_i$ be the set of $i$’s children. Then, for any node $c \in C_i$, we have

$$\left| D_{i,c} - \sum_{j \in T_c} d_j \right| \leq \sum_{j \in T_c} e_j,$$

where $d_j$ is the sensor reading at node $j$, $D_{i,c}$ is the data value maintained by node $i$ for node $c$, and $T_i$ is the subtree rooted at node $c$. Denote $i$’s parent node by $p$. According to the operation of an intermediate node presented above, we also have

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by node $p$ for node $i$. Therefore,

$$\left| D_{p,i} - d_i - \sum_{c \in C_i} D_{i,c} \right| \leq \sum_{c \in C_i} e_i,$$

where $D_{p,i}$ is the data value maintained by the base station for each of its child $i$. Let $C_i$ be the set of $i$’s children. Then, for any node $c \in C_i$, we have

$$\left| D_{i,c} - \sum_{j \in T_c} d_j \right| \leq \sum_{j \in T_c} e_j.$$
time of instance \( Q \) is polynomial to the size of instance \( P \). Next, we show that, for any integral bound \( K \), there exists a subset of the objects fitting in the knapsack with a total profit at least \( K \) for instance \( P \) if and only if there exists a feasible candidate precision allocation with highest normalized energy consumption rate at most \((ny - K)/z\) for instance \( Q \).

Note that all sensor nodes in instance \( Q \) have the same energy cost to send or receive a data update. This implies the energy consumption rates of the leaf nodes in Figure 2 (i.e., nodes \( 2, 3, \ldots, n \)) cannot exceed that of node 1. Also note that all sensor nodes have the same amount of residual energy. Therefore, regardless of precision allocation, the highest normalized energy consumption rate over all nodes is always that of node 1. It is then easy to establish a one-to-one correspondence between the object subsets fitting in the knapsack in instance \( P \) and the feasible candidate precision allocations in instance \( Q \). In fact, for any object subset \( S \), if \( S \) fits in the knapsack (i.e., \( \sum_{i \in S} c_i \leq C \)), the corresponding candidate precision allocation \( \{e_{i,2} : i \in S\} \cup \{e_{i,1} : i \notin S\} \) is feasible and in this case, node 1 has a normalized energy consumption rate \((ny - \sum_{i \in S} w_i)/z\), where \( \sum_{i \in S} w_i \) is the total profit of the object subset \( S \). Vice versa, for any feasible candidate precision allocation \((e_{1,x}, e_{2,x}, \ldots, e_{n,x})\), node 1 has a normalized energy consumption rate \((ny - \sum_{i=1}^{n} w_i)/z\), and the corresponding object subset \( \{i : x_i = 2\} \) fits in the knapsack. Thus, there exists an object subset with a total profit at least \( K \) for instance \( P \) if and only if there exists a candidate precision allocation with highest normalized energy consumption rate at most \((ny - K)/z\) for instance \( Q \).

Hence, the theorem is proven.

In the following, we present a distributed algorithm to compute a suboptimal candidate precision allocation. We advocate distributed algorithms because having all nodes reporting the estimated update rates \( u_{i,j} \)'s and residual energy levels \( p_i \)'s to the base station places further burdens of energy consumption on the nodes closer to the base station which are usually the energy bottlenecks in multi-hop networks.

To facilitate presentation, we shall refer to the sum of the local error bounds at the sensor nodes in the subtree \( T_i \) rooted at node \( i \) as its gross error bound. In addition to the candidate local error bounds, each sensor node \( i \) also selects a list of \( m \) thresholds \( T_{i,1} < T_{i,2} < \cdots < T_{i,m} \) for its gross error bound to assist the computation. For each threshold \( T_{i,j} \), node \( i \) computes a locally best precision allocation (we shall call it \( A_{i,j} \)) among \( i \) and its children under the constraint that the gross error bound at node \( i \) does not exceed \( T_{i,j} \). The computation in our algorithm is carried out in a bottom-up manner from the leaf sensor nodes to the base station. On computing the allocations \( A_{i,j} \)'s, each node \( i \) sends a candidate report message including a list \((E_{i,1}, U_{i,1}, R_{i,1}), (E_{i,2}, U_{i,2}, R_{i,2}), \ldots, (E_{i,m}, U_{i,m}, R_{i,m})\) to its parent, where \( E_{i,j} \) is the gross error bound at node \( i \) under \( A_{i,j} \) (it is straightforward that \( E_{i,1} \leq E_{i,2} \leq \cdots \leq E_{i,m} \)), \( U_{i,j} \) is the rate of data updates sent by node \( i \) to its parent under \( A_{i,j} \), and \( R_{i,j} \) is the highest normalized energy consumption rate of the sensor nodes in subtree \( T_i \) under \( A_{i,j} \). A parent node performs the local computation after receiving the candidate report messages from all children.

If \( i \) is a leaf sensor node, the thresholds are set the same as its candidate local error bounds, i.e., \( T_{i,j} = e_{i,j} \). Thus, \( E_{i,j} \)'s are simply \( e_{i,j} \)'s, and \( U_{i,j} \)'s are simply the estimated update rates \( u_{i,j} \)'s. Since \( i \) does not receive data updates from any other node, \( R_{i,j} \)'s are simply \( R_{i,j} = \frac{w_{i,j}}{p_i} \).

If \( i \) is an intermediate sensor node, it collects the candidate report messages from all of its children. Together with the locally estimated update rates \( u_{i,1}, u_{i,2}, \ldots, u_{i,m} \), node \( i \) computes a locally best precision allocation for each threshold \( T_{i,j} \) using Algorithm 2. Given a threshold \( T_{i,j} \), only the candidate local error bounds \( e_{i,h} \) satisfying \( e_{i,h} + \sum_{c \in C_i} E_{c,1} \leq T_{i,j} \) are likely to appear in a feasible precision allocation (step 3, otherwise the gross error bound at node \( i \) would exceed \( T_{i,j} \)). For each of these \( e_{i,h} \)'s, the best allocation of \( T_{i,j} - e_{i,h} \) among \( i \)'s children is computed using Algorithm 1 (step 4).

Algorithm 2 Locally Best Precision Allocation at Node \( i \) (in a Multi-Hop Network)

**Input:**
- \( T_{i,j} \): a threshold for the gross error bound of node \( i \)
- \( e_{i,*}, u_{i,*}, \cdots \): candidate local error bounds of node \( i \) and estimated data update rates to \( i \)'s parent node
- \( E_{c,*}, U_{c,*}, R_{c,*} \): gross error bounds, data update rates and highest normalized energy consumption rates received from each child \( c \) of node \( i \)

**Output:**
- \( A_{i,j} \): the computed best allocation, which includes the local error bound \( e_{i,y} \) allocated to node \( i \) and the gross error bound \( E_{c,x} \) allocated to each child \( c \) of node \( i \)
- \( E_{i,j}, R_{i,j} \): highest normalized energy consumption rate of the nodes in subtree \( T_i \) under \( A_{i,j} \)

1: \( R_{i,j} = +\infty; \)
2: for \( h = 1 \) to \( m \) do
3: if \( e_{i,h} + \sum_{c \in C_i} E_{c,1} \leq T_{i,j} \) then
4: compute the optimal candidate precision allocation for error bound \( T_{i,j} - e_{i,h} \) among \( i \)'s children using Algorithm 1 based on \( E_{c,*} \) and \( R_{c,*} \);
5: for each child \( c \) of \( i \), let \( E_{c,x} \) be the error bound of \( c \) in the optimal allocation, then \( U_{c,x} \) is the corresponding data update rate from \( c \) to \( i \), and \( R_{c,x} \) is the corresponding highest normalized energy consumption rate of the nodes in subtree \( T_c \);
6: \( R_i = \max \left( \frac{u_{i,h} + \sum_{c \in C_i} U_{c,x} \cdot v_i}{p_i}, \max_{c \in C_i} R_{c,x} \right); \)
7: if \( R_i < R_{i,j} \) then
8: \( R_{i,j} = R_i; \)
9: \( U_{i,j} = u_{i,h}; \)
10: \( E_{i,j} = e_{i,h} + \sum_{c \in C_i} E_{c,x}; \)
11: \( y_i = h; \)
12: for each child \( c \) of \( i \), \( y_c = x_c; \)
13: end if
14: end if
15: end for
Suppose \( E_{c,x} \) is the gross error bound of each child \( c \) in the best allocation. Then, \( U_{c,x} \) is the corresponding data update rate from \( c \) to \( i \), and \( R_{c,x} \) is the highest normalized energy consumption rate of the nodes in subtree \( T_c \). On computing \( i \)’s energy consumption rate, the highest normalized energy consumption rate of the nodes in subtree \( T_c \) can then be computed (step 6). The candidate local error bound \( e_{i,j} \) complexity that leads to the minimum highest energy consumption rate is included in the locally best precision allocation \( A_{c,j} \) (steps 7 to 13).

The corresponding allocation \( E_{c,y} \)’s among \( i \)’s children are also recorded in \( A_{i,j} \). The worst-case time complexity of Algorithm 2 is \( O(m^2 \cdot |C_i|) \), where \( |C_i| \) is the number of \( i \)’s children. Node \( i \) records the computed best allocation \( A_{i,j} \) for each threshold \( T_{i,j} \), and sends a candidate report message including the list \( \langle E_{i,1}, U_{i,1}, R_{i,1} \rangle, \langle E_{i,2}, U_{i,2}, R_{i,2} \rangle, \ldots, \langle E_{i,m}, U_{i,m}, R_{i,m} \rangle \) to its parent node.

The base station, on receiving the candidate report messages from all of its children, computes a locally best precision allocation among the children using Algorithm 1. The computed error bounds are then sent to the sensor nodes for their adjustments in a top-down manner. The base station sends a precision allocation message to its children including the gross error bounds allocated to them. An intermediate sensor node, on receiving its allocated gross error bound, retrieves the stored corresponding best allocation which contains a local error bound and a set of gross error bounds for its children. The intermediate node applies the local error bound to its partial aggregate results and sends the gross error bounds to its children in a precision allocation message. A leaf sensor node, on receiving its allocated gross error bound, simply takes it as the local error bound. In case the total error bound in the precision allocation computed by the base station does not add up to \( E \) exactly, the leftover error bound is allocated to the node with the highest normalized energy consumption rate.6

Similar to adaptive precision allocation in a single-hop network, the candidate local error bounds of each sensor node and the thresholds for its gross error bound are exponentially spaced around its current local and gross error bounds respectively. Let \( e_i \) and \( E_i \) be the current local and gross error bounds of sensor node \( i \) respectively. Given the number of candidates \( m = 2k + 1 \), the candidate local error bounds are selected as

\[
\frac{1}{2} e_i, 3 e_i, \frac{3}{4} e_i, \ldots, \frac{2k-1}{2k} e_i, e_i, \frac{2k+1}{2k} e_i, \ldots, \frac{5}{4} e_i, \frac{3}{2} e_i,
\]

and the thresholds for \( i \)’s gross error bound are selected as

\[
\frac{1}{2} E_i, 3 E_i, \frac{3}{4} E_i, \ldots, \frac{2k-1}{2k} E_i, E_i, \frac{2k+1}{2k} E_i, \ldots, \frac{5}{4} E_i, \frac{3}{2} E_i.
\]

Like that in a single-hop network, we dynamically adapt the adjustment period to limit the energy overhead of adjustments at the most energy-consuming node by a portion \( \alpha \) of its energy budget. Note that at an adjustment in a multi-hop network, a sensor node \( i \) receives a candidate report message from each child and sends one to its parent. It also receives a precision allocation message from its parent and sends one to its children. Thus, the energy cost at node \( i \) due to an adjustment is \( s_i + s_i' + (|C_i| + 1) \cdot v_i \), where \( s_i \) and \( s_i' \) are the sending costs to the parent and children respectively, \( v_i \) is the receiving cost, and \( C_i \) is the set of \( i \)’s children.

At an adjustment, the energy consumption rate of node \( i \) is estimated by \( (N_c s_i + N_c v_i)/L' \), where \( N_c \) and \( N_c' \) are the numbers of data updates sent to \( i \)’s parent and received from \( i \)’s children respectively in the past adjustment period, and \( L' \) is the duration of the past adjustment period. Node \( i \) suggests the duration of next adjustment period \( L_i \) as

\[
L_i = \frac{L' (s_i + s_i' + (|C_i| + 1) \cdot v_i)}{\alpha (N_c s_i + N_c' v_i)}.
\]

Each leaf node includes the suggested period in the candidate report message sent to its parent. Each intermediate node, on receiving the candidate report messages from its children, chooses the shortest period among that suggested locally and those received from its children. This shortest period is then included in the candidate report message sent by the intermediate node to its parent. Among all suggested periods received, the base station selects the shortest one \( L \) as the next adjustment period so as to cap the adjustment overhead at a portion \( \alpha \) of the energy consumed at the most consuming node. \( L \) is then included in the precision allocation messages sent to all sensor nodes.

VI. PERFORMANCE EVALUATION

A. Experimental Setup

We developed a simulator based on ns-2 [37] and NRL’s sensor network extension [38] to evaluate the proposed adaptive precision allocation scheme. We used the following energy models [10]. The energy consumed by a sensor node to send a message is \( s \cdot (\alpha + \beta \cdot d^2) \), where \( s \) is the message size, \( \alpha = 50 \) nJ/b is a distance-independent term, \( \beta = 100 \) pJ/bm² is the coefficient for a distance-dependent term, \( q = 2 \) is the exponent for the distance-dependent term, and \( d \) is the transmission distance. The energy consumed by a sensor node to receive a data update is \( s \cdot \gamma \), where \( \gamma = 50 \) nJ/b is a coefficient independent of transmission distance. In our experiments, the default message size was set at 48 bytes [16]. The initial energy budget at each sensor node was set at 0.5 Joule.

We simulated a single-hop network of 10 sensor nodes and multi-hop networks of 100 sensor nodes. The layout of the single-hop network is shown in Figures 3. The multi-hop network topologies were generated by randomly placing the base station and 100 sensor nodes in a 200m×200m area. To simulate the spatial irregularity in sensor network deployment [39], we divided the area into a 4×4 grid. The probabilities of deploying sensor nodes in the grid cells were assumed to follow a Zipf-like distribution. That is, the 16 grid cells were randomly ordered into a list and the probability to deploy sensor nodes in the \( i \)th cell on the list was set to \( i^{-\theta} / c \), where \( \theta \geq 0 \) is the Zipf parameter and \( c = \sum_{i=1}^{16} i^{-\theta} \) is a normalization factor [40]. The default value of \( \theta \) was set at

5The receiving cost is normally independent of the sender [10], [11], [33].
1. The sensor nodes were assumed to have a maximum radio transmission range of 40m. If two sensor nodes were within the radio range of each other, they were considered neighbors in the network connectivity graph. The breadth first search tree rooted at the base station was then computed from the connectivity graph and used as the routing infrastructure for data collection [21], [27]. We have experimented with many randomly generated network topologies and observed similar performance trends. Due to space limitations, we shall only report the results of a sample network topology in this paper. The layout of the topology is shown in Figure 4, where the solid circle represents the base station, the remaining circles represent the sensor nodes and the lines represent the links in the routing tree.

Fig. 3. The Single-Hop Network Layout.

Fig. 4. Sample Multi-Hop Network Layout.

We made use of the data provided by the LEM project [41] at the University of Washington to simulate the physical phenomena in the immediate surroundings of sensor nodes. Weather data were collected in the LEM project from several stations in the Washington and Oregon states. We used the temperature (TEMP) and solar radiation (SOLAR) traces logged by the station at the University of Washington from August 2004 to August 2005 in our experiments. Each trace consisted of more than 500,000 readings captured at a sampling period of 1 minute. Figure 5 shows some representative segments of these traces. The TEMP and SOLAR data both fluctuate over time – their readings are higher in the daytime and lower at night. In particular, the SOLAR readings remain unchanged regularly because the solar radiation is 0 at night. For each of the TEMP and SOLAR traces, we extracted 100

different subtraces starting at randomly selected timepoints and associated them with the sensor nodes in our simulated network. The sampling period between two successive readings in the trace was assumed to be 1 time unit.

The base station computes the AVERAGE aggregation of the readings collected from all sensor nodes with a designated error bound $E$. As discussed in Section III, in this case, the total error bound allocated to the sensor nodes should be capped by $n \cdot E$, where $n$ is the number of sensor nodes. The experiments started with the error bound uniformly allocated to the sensor nodes, i.e., each node was allocated an error bound of $E$. The following precision allocation schemes were simulated for performance comparison. We measured the energy consumption of each sensor node and the network lifetime in the experiments.

- **Our Adaptive Precision Allocation (Adaptive-PA):** This is the adaptive precision allocation scheme proposed in Sections IV-C and V-B. By default, each sensor node selected $m = 7$ candidate error bounds and the energy cost due to adjustments was capped at $\alpha = 0.2\%$ of the energy consumed at the most consuming node. The performance impacts of $m$ and $\alpha$ are investigated in Section VI-B. We assumed that each data value in the message (e.g., sensor reading and candidate error bound) took up 2 bytes. In addition, a timestamp of 2 bytes was included in all messages for ordering and synchronization purposes. The largest messages encountered in our experiments were the candidate report messages in multi-hop networks. Recall that the candidate report message includes a list of $\langle E_{i,x}, U_{i,x}, R_{i,x}\rangle$’s and a suggested next adjustment period. It requires a total of $6m + 4 = 46$ bytes when $m = 7$, which fits into the default message size.

- **Uniform Precision Allocation (Uniform-PA):** The error bound is evenly partitioned among all sensor nodes [7], i.e., the precision allocation remains the initial one. This is a simple and static scheme which does not differentiate the sensor nodes by the changing pattern of sensor readings, the residual energy, and the communication cost with the base station.

- **Burden-based Precision Allocation (Burden-PA):** Olston et al. [6] presented a burden-based precision allocation scheme for aggregate queries over distributed data streams. Their objective was to minimize the total
communication cost between data sources and the data sink. In our experiments, the energy consumed by each sensor node to send a data update to its parent was taken as a measure of its communication cost.\(^5\) Burden-PA works by periodically reducing the error bound of each sensor node by a shrink percentage and redistributing the leftover portion among the sensor nodes. As suggested by [6], the shrink percentage was set at 5%. We simulated Burden-PA over a wide range of different adjustment periods (from 144 to 2880 time units, which correspond to 0.1 to 2 days of data traces) and found that no single period provided the best performance for all experimental settings. Thus, to favor Burden-PA, for each experimental setting, we selected the best result obtained over all adjustment periods tested and present it in this paper.

- **Potential-Gain-based Precision Allocation (PGain-PA):** To reduce the total number of messages in the network, Deligiannakis et al. [9] presented a precision allocation scheme for sensor data aggregation based on online estimation of potential gains. Similar to Burden-PA, PGain-PA periodically reduces the error bound of each sensor node by a shrink percentage and redistributes the leftover portion among the sensor nodes. As suggested by [9], the shrink percentage was set at 40%. Again, we simulated PGain-PA over a wide range of adjustment periods (from 144 to 2880 time units) and selected the best result obtained to present in this paper.

### B. Effect of \(m\) and \(\alpha\) in Adaptive-PA

![Network Lifetime vs. Number of Candidate Error Bounds](image)

**Fig. 6.** Network Lifetime vs. Number of Candidate Error Bounds in Adaptive-PA (TEMP Trace, \(E = 0.6\ F\)).

First, we investigate the performance impact of the number of candidate error bounds \(m\) in the proposed Adaptive-PA scheme. Figure 6 shows the network lifetime for different \(m\) values when the error bound \(E\) was set at 0.6 for the TEMP trace.\(^9\) Note that when \(m = 1\), the current error bound is the only candidate. Thus, the optimal candidate precision allocation computed by Algorithm 1 is always the same as the current allocation. Since the experiments started with uniformly allocated error bounds, Adaptive-PA degenerates to Uniform-PA at \(m = 1\). The flexibility of precision allocation increases with \(m\). As seen from Figure 6, an \(m\) value of 3 improves network lifetime significantly compared to \(m = 1\) (by factors of 3.4 and 1.7 in single-hop and multi-hop networks respectively). The network lifetime is generally insensitive to \(m\) when \(m\) exceeds 5. Since the largest allowable \(m\) for a candidate report message to fit into the default message size is 7, \(m\) was set at 7 in the remaining experiments.

Recall that Adaptive-PA limits the energy overhead of adjustments at the most energy-consuming node by a portion \(\alpha\) of its energy budget. The setting of \(\alpha\) reflects a tradeoff between overhead and adaptivity, both of which increase with \(\alpha\). Figure 7 shows the network lifetime for different \(\alpha\) values. As expected, the curve of network lifetime is convex for most system configurations tested. In general, the performance of Adaptive-PA is not very sensitive to the \(\alpha\) value from 0.1% to 0.5%. Therefore, we shall report only the experimental results for the default \(\alpha = 0.2\%\) in the remainder of this paper.

### C. Performance Comparison in Single-Hop Networks

Figure 8 shows the network lifetime as a function of the designated error bound \(E\) on data aggregation for different precision allocation schemes in the single-hop network of Figure 3. Note that an error bound \(E = 0\) implies exact data aggregation (the leftmost points in Figure 8). With exact data aggregation, all sensor nodes must be allocated error bounds of 0. Therefore, in this case, the four precision allocation schemes have similar performance.

As seen from Figure 8, the network lifetime increases with error bound. When \(E > 0\), the proposed Adaptive-PA scheme significantly outperforms the other schemes for both traces tested. Even if the readings at all sensor nodes follow similar changing patterns, it is not desirable to allocate the same error bound to all nodes because they are geographically distributed. In a single-hop network, a node farther away from the base station consumes more energy in sending a data update than a node closer to the base station. Among the four precision allocation schemes examined, Uniform-PA and PGain-PA do not take this heterogeneity into consideration. Thus, as shown in Figure 8, Adaptive-PA improves network lifetime by factors up to 3.4 and 2.6 compared to Uniform-PA and PGain-PA respectively. To show the importance of balancing energy consumption in extending network lifetime, we plot in Figure 9 the total energy consumed by each sensor node by the time when the first node runs out of energy (i.e., the network lifetime elapsed). Under Adaptive-PA, most nodes were close to exhausting their energy when the network lifetime elapsed. However, under Uniform-PA and PGain-PA, the nodes close to the base station (i.e., nodes 3 and 8 in Figure 3) consumed as low as 5% ~ 20% of the energy budget only.

Burden-PA considers the heterogeneity in communication cost due to transmission distance. However, the objective of Burden-PA is to minimize the total communication cost. Figure 8 shows that our Adaptive-PA scheme extends network...
lifetime by a factor up to 1.9 over Burden-PA. This implies minimizing network-wide total energy consumption does not necessarily balance the energy consumption of the sensor nodes. As seen from Figure 9(b), under Burden-PA, nodes 3 and 8 consumed as low as 12% and 36% of the energy respectively when the network lifetime elapsed.

**D. Performance Comparison in Multi-Hop Networks**

We have implemented in-network aggregation in the experiments for multi-hop networks. Figure 10 shows the results for the multi-hop network of Figure 4. The performance trends remain similar to those in the single-hop network. The network lifetime increases rapidly with error bound. For example, under Adaptive-PA, increasing \( E \) from 0 (exact data aggregation) to 0.2 and 20 prolongs the network lifetime by factors of 2.6 and 3.8 for the TEMP and SOLAR traces respectively. This demonstrates the effectiveness of approximate data aggregation in improving energy efficiency.

Comparing the performance of different precision allocation schemes, Adaptive-PA significantly outperforms the other schemes for both traces tested. As seen from Figure 10, the
improvements over Uniform-PA, Burden-PA and PGain-PA are up to factors 3.7, 1.6 and 1.5 respectively. Comparing Figures 8 and 10, it is also observed that the relative performance of PGain-PA to Burden-PA improves in the multi-hop network. This is because PGain-PA takes into account the topological relations among the sensor nodes as well as in-network aggregation. In contrast, Burden-PA treats the sensor nodes independently and does not model in-network aggregation. However, PGain-PA aims at minimizing the total number of messages transmitted in the network without considering the heterogeneity in communication cost. Neither does it attempt to balance the energy consumption at different nodes. Thus, its performance is still much worse than our Adaptive-PA.

Figure 11 shows the distribution of energy consumed at all sensor nodes when the network lifetime elapsed. A point \((x, y)\) on the curve means that \(x\)% of the nodes consume more than \(y\) Joule energy each. It is clear that by balancing the energy consumption at different nodes, the proposed Adaptive-PA scheme makes much better utilization of the energy budgets than the other schemes. For the TEMP trace (Figure 11(a)), over 30% of the nodes consume more than 80% of the energy budget (i.e., 0.4 Joule) in Adaptive-PA, while only 2%, 4% and 3% of the nodes do so in Uniform-PA, Burden-PA and PGain-PA respectively. This helps Adaptive-PA to improve network lifetime over the other three schemes. Similar trends are observed for the results of the SOLAR trace (Figure 11(b)).

VII. CONCLUSION

We have investigated adaptive precision allocation to extend the lifetime of data aggregation with precision guarantees in wireless sensor networks. The purpose of precision allocation is to differentiate the quality of data collected from different sensor nodes, thereby balancing their energy consumption. Our proposed schemes effectively exploit the tradeoff between data quality and energy consumption. These schemes dynamically adjust the error bounds allocated to the sensor nodes. The basic scheme for single-hop networks is based on the analysis of an optimal precision allocation in terms of network lifetime. The extended scheme for multi-hop networks takes into consideration the topological relations among the sensor nodes as well as the effect of in-network aggregation. Experimental results using real data traces show that: (1) tolerating just a small degree of inaccuracy in data collection prolongs network lifetime substantially; (2) due to geographically distributed nature of sensor networks, uniform precision allocation does not perform well even if the readings at all sensor nodes follow similar changing patterns; (3) to extend network lifetime, it is more important to balance the energy consumption of the sensor nodes than to minimize network-wide total energy consumption; (4) the proposed adaptive precision allocation schemes significantly outperform existing methods over a wide range of system configurations.
REFERENCES


