A Modified Coefficient Decimation Method to Realize Low Complexity FIR Filters with Enhanced Frequency Response Flexibility and Passband Resolution

Abhishek Ambede, Smitha K. G. and A. P. Vinod

Abstract—Low complexity, reconfigurable finite impulse response (FIR) filters using coefficient decimation method (CDM) has been recently proposed in literature. In this paper, we propose a modified coefficient decimation method (MCDM) which enhances the flexibility of CDM in obtaining FIR filters with varied passband locations. The resolution of the center frequency locations in the multi-band frequency responses obtained using MCDM is twice that of the conventional CDM. Further, the stopband attenuation of FIR filters realized using our MCDM is higher compared to the filters obtained using conventional CDM. Also, for the same prototype modal (original) filter, the number of distinct frequency band locations that can be obtained after coefficient decimation is greater for MCDM than CDM due to the increased center frequency resolution of the former method. The hardware realization architecture of MCDM is presented. The advantages of our method for realizing reconfigurable filter banks are also discussed.

Keywords—Finite impulse response (FIR) filters, low complexity, reconfigurability, coefficient decimation.

I. INTRODUCTION

FINITE impulse response (FIR) filters find extensive applications in digital signal processing. FIR filters are preferred over IIR filters in communication systems due to their characteristics like inherent stability and linear phase. However, for the same stopband and transition band specifications, the complexity of a FIR filter would be higher than its IIR counterpart due to the requirement of increased filter order for the former than the latter. Many techniques have been proposed in literature to reduce the complexity of FIR filters.

In [1], a frequency response masking (FRM) approach was proposed to realize sharp transition band FIR filters. In this technique, the main principle is to use a set of lower order sub-filters to obtain the sharp filter responses. The main disadvantage of FRM is the arbitrary passband location of the resulting filters that cannot be absolutely controlled. Many approaches have been proposed that attempt to reduce the arithmetic complexity of FRM technique (reduction in number of multipliers in [2], elimination of a masking filter in [3]), but the abovementioned limitation of arbitrary passband location persists in these modified FRM techniques. In [4], a digit based reconfigurable FIR filter architecture was proposed. In this technique, the tap number and the number of non-zero digits in each tap are arbitrarily assigned and the aim is to reduce the filter complexity by reducing coefficient precision. This method has a high utilization of hardware resources which makes it infeasible for applications that need low power and low resource utilization.

In [5], the coefficient decimation method was proposed for realizing reconfigurable FIR filters. In CDM, if an N-tap FIR filter is decimated by M, i.e., if every Mth coefficient of the filter is kept unchanged and remaining coefficients replaced by zeros, a multiband frequency response is obtained with the center frequencies located at integer multiples of 2π/M. As M is increased, there is a decrease in complexity due to the replacement of filter coefficients by zeros. The number of coefficient multiplications required to implement the decimated filter is thus N/M. The passband width of the obtained frequency bands is the same as that of the modal (original prototype) filter which is decimated. The various frequency bands can then be separated using masking filters or subtracting one response from the other.

In [6], CDM is used to obtain flexible filter banks for channelization in software defined radios (SDRs). In this work, it is shown that the coefficient decimation based filter banks (CDFBs) have a higher degree of reconfigurability with respect to the passband widths and passband center frequency locations than the widely used discrete Fourier transform based filter banks (DFTFBs), and offer significant savings in resource utilization and speed. Also, CDFBs can receive channels (frequency bands) of multiple communication standards simultaneously which is not possible in the DFTFBs using the same filter bank. The complexity of CDFB is shown to be lower than other methods such as the per-channel (PC) approach, modulated perfect reconstruction bank (MPRB), and the filter bank based on Goertzel algorithm [6].

In this paper, we propose a modified CDM (MCDM). Our MCDM significantly improves the flexibility of the conventional CDM [5] (conventional CDM [5] is referred to as “CDM” in the rest of the paper) by providing a higher degree of resolution for center frequency selection. In our MCDM, the frequency bands can be obtained with a center frequency resolution of π/M whereas the achievable resolution is only 2π/M in CDM. Furthermore, when compared with CDM, the reconfigurable filters realized using our method have other advantages such as improved stopband,

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attenuation and better control over the frequency band selection. Also, the number of distinct frequency bands that can be obtained using our MCDM is larger than that of CDM.

The rest of the paper is organized as follows: Section II presents the mathematical formulation and design procedure of our MCDM. Section III presents a design example that illustrates the advantages of MCDM over CDM. An architecture for implementing filters based on MCDM is also presented. The conclusions are given in Section IV.

II. MODIFIED COEFFICIENT DECIMATION METHOD

In CDM [5], if the coefficients of an FIR filter are decimated by \( M \), i.e., if every \( M \)-th coefficient is retained and the others replaced by zeros, an FIR filter with a multi-band frequency response is obtained. The frequency response of the obtained filter will have bands with center frequencies at \( 2\pi k/M \), where \( k \) is an integer ranging from 0 to \( M-1 \). If \( H(e^{j\omega}) \) denotes the Fourier transform of the modal filter coefficients, then the Fourier transform of the modified coefficients is given by

\[
H'(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} H(e^{j(\omega - 2\pi k/M)})
\]  

(1)

The mathematical derivation for the same is given in [5].

In our modified coefficient decimation method (MCDM), we introduce a new coefficient decimation operation. In this, if the decimation factor is \( M \), then every \( M \)-th coefficient of the FIR filter is retained and the sign of every alternate retained coefficient is reversed. All other coefficients are replaced by zeros. As a result of this operation, an FIR filter with a multi-band frequency response is obtained having center frequencies at \( (2k+1)\pi/M \), where \( k \) is an integer ranging from 0 to \( M-1 \). The mathematical proof for this is given below:

Let the modal filter coefficients be denoted by \( h(n) \), and the modified coefficients be denoted by \( h'(n) \). Now,

\[
h'(n) = h(n).d_M(n)
\]  

(2)

where \( d_M(n) \) denotes a function that performs the operation of selecting the coefficients and performing sign changes. It can be represented as

\[
d_M(n) = \begin{cases} 
  1 & \text{for } n = mM, m = 0,2,4,6,... \\
  -1 & \text{for } n = pM, p = 1,3,5,7,... \\
  0 & \text{otherwise}
\end{cases}
\]  

(3)

Thus, \( d_M(n) \) is periodic with period \( 2M \). Its Fourier series expansion [7] is thus given by

\[
d_M(n) = \frac{1}{2M} \sum_{k=0}^{2M-1} D(k)e^{j2\pi kn/2M}
\]  

(4)

where \( D(k) \) represents complex Fourier series coefficients which are given by

\[
D(k) = \sum_{n=0}^{2M-1} d_M(n)e^{-j2\pi kn/2M}
\]  

(5)

From (3) and (5), we can derive that \( D(k) = 0,2,0,2,0,2,... \) for any value of \( M \), for \( k = 0 \) to \( 2M-1 \).

Thus, using the above observation and substituting for \( D(k) \), we can rewrite equation (4) as

\[
d_M(n) = \frac{1}{M} \sum_{k=0}^{M-1} e^{j(2k+1)n/M}
\]  

(6)

The Fourier transform of the modified coefficients is

\[
H'(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h'(n)e^{-j\omega n}
\]

Substituting from (2) and (6), we get

\[
H'(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) \left( \frac{1}{M} \sum_{k=0}^{M-1} e^{j(2k+1)n/M} \right) e^{-j\omega n}
\]

(7)

By interchanging the summations,

\[
H'(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} h(n)e^{-j(n(\pi/2k+1)/M)}
\]

\[
\therefore H'(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} H(e^{j(\omega - k\pi/2M)})
\]  

(8)

Thus, the frequency response of the modified coefficients is scaled by \( M \) and the original frequency spectrum is replicated at the locations \( (2k+1)\pi/M \), where \( k = 0 \) to \( M-1 \). Different multi-band frequency responses can thus be obtained for different values of \( M \). Also, to obtain the original magnitude response, the output of the filters is to be scaled by \( M \). The inferences from the above derivation are similar in nature to those of the CDM, the main difference being the change in the location of center frequencies.

From (1) and (8), we can see that using MCDM, FIR filters having multi-band frequency responses with a center frequency resolution of \( \pi/M \) can be obtained.

The design procedure of MCDM-based FIR filters for desired frequency responses is given below.

- Design a modal filter that meets the desired passband and stopband specifications. While designing the modal filter, the deterioration in stopband attenuation observed after coefficient decimation has to be considered and the filter order is to be selected accordingly. This deterioration is mathematically given by

\[
\delta_{s\text{(modal)}} = \frac{\delta_{s\text{(final)}}}{M}
\]  

(9)

where \( \delta_{s\text{(modal)}} \) is the stopband attenuation (SA) of the modal filter and is \( \delta_{s\text{(final)}} \) is the SA of the filter obtained after coefficient decimation by \( M \) [8]. Hence, we need to overdesign the modal filter to compensate for the deterioration in SA. This requirement is also applicable to CDM [5].

- Using (1) and (8), select the appropriate values of \( M \) that will lead to multi-band frequency responses containing the desired passband locations.
• Use the proposed MCDM for different \( M \) values to obtain frequency responses containing the desired passbands.
• From the resulting multi-band frequency responses, separate the desired frequency bands by subtracting the frequency responses from one another or by using low complexity frequency response masking filters that extract specific bands.

III. DESIGN EXAMPLE AND HARDWARE ARCHITECTURE

A. Design Example

The advantages of the MCDM over the CDM are discussed below with the help of a design example.

Fig. 1 (a) shows the frequency response of a lowpass modal filter designed such that the passband edge frequency \( f_p \) = 0.12, stopband edge frequency \( f_s \) = 0.125 (both frequencies normalized for range 0 to 1) and the SA = -50dB.

Figures 1(b) and 1(c) show the frequency responses obtained from the modal filter shown in Fig. 1(a), for \( M=2 \) using CDM and our MCDM respectively. Figures 1(d) and 1(e) show corresponding frequency responses for \( M=3 \). As can be seen in the figures for \( M=3 \), and also inferred from (1) and (8), the frequency responses obtained for odd values of decimation for CDM and MCDM are mirror images of each other.

From (1) and (8), it is clear that for even values of \( M \), the desired frequency band can be obtained after decimating the modal filter by \( M/2 \) using MCDM instead of decimating by \( M \) using the CDM. This can be seen in Fig. 1(b), Fig. 1(c) and Fig. 1(f) which show the frequency responses at \( M=2 \) using MCDM and \( M=4 \) using CDM respectively. The inherent drawback of coefficient decimation (both CDM and MCDM) is that the stopband attenuation deteriorates with higher decimation factors (\( M \) values) as given by (9), which can be observed from the frequency responses of coefficient decimated filters shown in Fig. (1). However, when MCDM is employed, the desired frequency response (passband locations) can be obtained using a smaller decimation factor whereas a larger decimation factor would be required to obtain the same desired passband location using CDM. The use of smaller decimation factor in MCDM thus has less impact on SA deterioration compared to CDM because SA deterioration increases with decimation factor. Also, as lower value of decimation means lesser SA, the modal filter of MCDM can be designed with a lower order than that required in CDM, to meet the same desired SA specification for the final filters obtained after coefficient decimation. This in turn

Fig. 1(a). Frequency response of the modal (prototype) filter.

Fig. 1(b). Frequency response of the filter obtained using CDM for \( M=2 \).

Fig. 1(c). Frequency response of the filter obtained using MCDM for \( M=2 \).

Fig. 1(d). Frequency response of the filter obtained using CDM for \( M=3 \).

Fig. 1(e). Frequency response of the filter obtained using MCDM for \( M=3 \).

Fig. 1(f). Frequency response of the filter obtained using CDM for \( M=4 \).
will reduce the complexity and hardware resource utilization of MCDM.

Fig. 1(f) shows the frequency response of the filter obtained using CDM for $M=4$. If the frequency band centered at 0.5 needs to be separated, a masking filter is to be used [6], or the response in Fig. 1(b) has to be subtracted from the one in Fig. 1(f). Obtaining this band separately as shown in Fig. 1(c) using MCDM involves no computation overhead of masking filter or spectral subtraction as opposed to CDM.

Due to the improvement in center frequency resolution from $2\pi/M$ in CDM to $\pi/M$ in MCDM, the flexibility in obtaining different frequency bands increases significantly. Also, some frequency bands that cannot be obtained using CDM can be obtained using the MCDM. For example, to obtain a frequency band centered at 0.1667 using the modal filter in Fig. 1(a), the theoretical value of $M$ required to be used in CDM is 12. However, it is not possible to use $M=12$ in CDM as values of $M$ beyond $1/f_s$ (i.e., $M > 8$) would lead to aliasing. On the other hand, using MCDM, we can obtain the above band centered at 0.1667 when the value of $M$ is 6 (which is less than the maximum permissible alias-free value $M$, i.e., 8). This increased flexibility can be significantly advantageous while designing multi-channel filter banks for communication systems.

B. Hardware Realization Architecture

Fig. 2 shows the proposed architecture for implementing the MCDM. It consists of 2:1 multiplexers which are used to retain a filter coefficient or to make it zero, according to the value of $M$. The sign reversal of the corresponding coefficients is performed by the adder/subtractor (add/sub) blocks. This is valid because $(-h) \otimes x = -(h \otimes x)$, where $h$ is filter coefficient and $x$ is input. To obtain the frequency response of the modal filter, all the coefficients are retained and the add/sub blocks are used as adders. Also, when CDM is to be employed, every $M^{th}$ coefficient is retained by the multiplexers and the others are made zero. The add/sub blocks are used as adders in this case.

The resulting output after applying MCDM needs to be scaled by $M$ to obtain the original magnitude response. This can be done by multiplying the output of the filter by $M$ as shown in the Fig. 2, as $(M \times h) \otimes x = M \times (h \otimes x)$. If specific frequency bands are to be separated, different multi-band frequency responses can be subtracted from each other or appropriate masking filters can be applied to the scaled output. Similarly, an architecture can be derived from the one shown in Fig. 2 to obtain different coefficient decimation responses in parallel. If complementary responses of the decimated responses are also obtained, then all of these can be algebraically operated upon to get highly reconfigurable and flexible filter banks.

IV. CONCLUSION

We have proposed a modified coefficient decimation method (termed ‘MCDM’) that increases the flexibility of the conventional coefficient decimation method (‘CDM’). The resolution ($\pi/M$) of the locations of the center frequencies of passbands in MCDM is twice that of the CDM. This in turn provides a higher degree of reconfigurability of the FIR filters that can be obtained using MCDM. Also, it is shown that the MCDM can be used to obtain better stopband attenuation from the derived filters compared to CDM, along with a reduction in complexity resulting due to the use of a lower order modal filter. The advantages of using MCDM to design low complexity and flexible filter banks are also discussed.

REFERENCES