Efficient GA-Based Electromagnetic Optimization Using HDMR-Generated Surrogate Models

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Abstract—An efficient surrogate modelling technique that supports the genetic algorithm (GA) based optimization of electromagnetic (EM) devices is presented. The proposed method leverages high dimensional model representation (HDMR) expansions, which approximate observables or objective functions as finite sums of “component functions” that represent independent and combined contributions of design variables to the observables or objective functions; this feature of the proposed method addresses concern (i) above [1]. The HDMR expansions are approximated via an ME-PC method [3]. The ME-PC method effectively tailors the sampling points used for polynomial approximation to the rate of variation of the function being approximated; this feature of the proposed method addresses concern (ii) above. Once accurate surrogate models have been built, the proposed method runs a classical GA to search the multidimensional design space for optimal designs.

I. INTRODUCTION

In recent years, GAs have been successfully applied to the design of a wide range of EM devices, from frequency selective surfaces to waveguides and antennas. While canvassing multimodal design spaces to find optimal designs, GAs often require the evaluation of objective functions for a large number of design candidates. This limits their applicability to the design of electrically large EM systems, for which the evaluation of each objective function requires the execution of a CPU-intensive full-wave EM simulator. Therefore, the GA-based optimization of EM devices often relies on surrogate modeling methods to accelerate the evaluation of pertinent observables or objective functions. Unfortunately, surrogate models often lack accuracy, especially (i) when the dimensionality of the design space is high and/or (ii) when the pertinent observables or objective functions exhibit rapid variations.

Here, a new technique to construct surrogate models for observables or objective functions pertinent to the optimization and design of EM devices is presented. The proposed method leverages HDMR expansions to express pertinent observables or objective functions as finite sums of “component functions” that represent independent and combined contributions of design variables to the observables or objective functions; this feature of the proposed method addresses concern (i) above [1]. The HDMR expansions are constructed iteratively by including only the “most significant” component functions to minimize the computational cost of building the surrogate models [2]. The component functions featured in HDMR expansions are approximated via an ME-PC method [3]. The ME-PC method effectively tailors the sampling points used for polynomial approximation to the rate of variation of the function being approximated; this feature of the proposed method addresses concern (ii) above. Once accurate surrogate models have been built, the proposed method runs a classical GA to search the multidimensional design space for optimal designs.

II. FORMULATION

Let \( \mathbf{x} = (x_1, x_2, \ldots, x_{N_{\text{dof}}}) \) be an \( N_{\text{dof}} \)-dimensional vector defined over the design space \( \Omega \). The elements of \( \mathbf{x} \), \( x_i \), \( i = 1, \ldots, N_{\text{dof}} \), are real design variables in a complex system’s geometry and configuration (e.g., locations of antennas) and/or excitation (e.g., phases and magnitudes of voltage source). Let \( V(\mathbf{x}) \) represent the observable (e.g., the side lobe level (SLL) of a radiation pattern, or a voltage standing wave ratio (VSWR) on an antenna feed) that can only be evaluated via a CPU-intensive full wave EM simulator. Oftentimes, the execution of a GA requires the evaluation of a large number of \( V(\mathbf{x}) \) values, which renders its direct application to the design of electrically-large EM devices infeasible. To circumvent this difficulty, one may apply a GA to an approximate representation of \( V(\mathbf{x}) \), henceforth called a surrogate model. In this case, the difficulty lies in inexpensively generating an accurate but cheap to evaluate surrogate model of \( V(\mathbf{x}) \). Here, this is achieved via an HDMR expansion, which approximates \( V(\mathbf{x}) \) in terms of component functions as [1]

\[
V(\mathbf{x}) = V_0 + \sum_{i=1}^{N_{\text{dof}}} V_i(x_i) + \sum_{i<j}^{N_{\text{dof}}} V_{ij}(x_i, x_j) + \sum_{i<j<k} V_{ijk}(x_i, x_j, x_k) + \cdots + V_{N_{\text{dof}}}(x_1, \ldots, x_{N_{\text{dof}}})
\]

(1)

where \( V_0 \) is the zeroth-order component function which is constant over \( \Omega \); \( V_i(x_i) \) denotes a first-order component function that represent the individual contributions of each \( x_i \) to \( V(\mathbf{x}) \); \( V_{ij}(x_i, x_j) \) represents a second-order component function that reveals the combined contributions of \( x_i \) and \( x_j \) to \( V(\mathbf{x}) \); the remaining terms in (1) are higher order combined contributions of design variables to \( V(\mathbf{x}) \). The representation in (1) can be constructed using the cut-HDMR method [1].
which expresses the component functions in terms of observable values on lines, planes, and hyperplanes (i.e. cuts) in the $N_{dof}$-dimensional design space. The observable values on cuts are approximated using the adaptive ME-PC method described in [3).

The HDMR construction and its efficiency perhaps are best described by an example. Assume $N_{dof} = 3$; the representation in (1) can be expanded as

$$V(x) = V_0 + V_1(x_1) + V_2(x_2) + V_3(x_3) + V_{12}(x_1, x_2) + V_{13}(x_1, x_3) + V_{23}(x_2, x_3). \quad (2)$$

Assume that the observable consists of a constant term and monomials, i.e., $V(x) = 1 + x_1^2 + x_2^2 + x_3^2$. The component functions $V_0$, $V_i(x_i)$, $V_{ij}(x_i, x_j)$, and $V_{ijk}(x_i, x_j, x_k)$ in (2) are needed to approximate $V(x)$. The remaining component functions in (2) are redundant and should be excluded from the HDMR expansion. Including only the “most significant” component functions in HDMR expansion significantly reduces the computational cost of HDMR construction (and thereby surrogate model generation). An iterative scheme [2] that excludes all but the most relevant component functions in the HDMR expansion starts by computing the weights

$$\beta_i = \int V_i(x_i) dx_i/\sqrt{V_0}; \quad i = 1, \ldots, N_{dof}. \quad (3)$$

These weights are measures of the contributions of each first order component function’s mean to the overall mean computed at the zeroth level. If $\beta_i$ is larger than a prescribed tolerance $\epsilon$, then the component functions (or design variables) pertinent to $\beta_i$ are marked as “important”. The second-order component functions involving these “important” design variables are marked as “candidates” for constructing the HDMR expansion at the second level. They are only added to the expansion if their weights are larger than $\epsilon$; their weights are computed by taking the ratio of each “candidate” second-order component function’s mean to the overall mean computed at the first level (as in (3)). This scheme is repeated in an iterative manner for all levels. The HDMR expansion is assumed to have converged if the decay rate of the relative difference between the observable means computed at two consecutive levels is smaller than a given tolerance $\epsilon$.

III. NUMERICAL RESULTS

The proposed method is applied to the design of a linear array of stacked patch antennas [Fig. 1(a),(b)]. The stacked patch antenna composed of 8 mm thick upper/lower substrates with relative dielectric constants of 2.40/3.27 and loss tangents of 0.0012/0.0020, respectively. The antenna operates in a frequency band from 1.14 GHz to 1.26 GHz. The linear array consists of ten antenna elements [Fig. 1(c)]. The design variables are the locations of antenna elements along y-axis, $x_1$, $x_2$, $x_3$, $x_4$, $x_5$, $x_6$, $x_7$, $x_8$, and $x_9$, ($N_{dof} = 10$); that can vary in ranges $[-50,50]$ mm, $[150,250]$ mm, $[350,450]$ mm, $[550,650]$ mm, $[750,850]$ mm, $[950,1050]$ mm, $[1150,1250]$ mm, $[1350,1450]$ mm, $[1550,1650]$ mm, $[1750,1850]$ mm, respectively. The design goals are to achieve $\text{SLL} \leq -15 \text{ dB}$ and active VSWR $\leq 2$, $i = 1, \ldots, 10$, $k = 1, \ldots, N_{freq}$, where $N_{freq} = 21$ is the number of equally-spaced frequency samples. The surrogate models of SLLs and active VSWRs are generated using iterative cut-HDMR with $\epsilon_{1} = 10^{-2}$, $\epsilon_{2} = 10^{-6}$, and a maximum HDMR order of 2. The proposed method required 3625 deterministic simulations to generate surrogate models with maximum L-2 norm error of $3.45 \times 10^{-3}$. Once the surrogate models are generated, a classical GA is run to find the optimum design in $\Omega$. Active VSWRs and SLLs of the GA-synthesized array are compared with those of a uniformly spaced array with inter-element spacing of 200 mm [Fig. 1(d)-(n)]. Apparently, the proposed method finds a design that meets the specified design goals. Synthesizing this large array using a GA by relying on full-wave EM simulations would be impractical compared to the proposed method.

REFERENCES


Figure 1. (a) Stacked patch antenna geometry, (b) Dimensions of patches (in mm), (c) Linear array of ten stacked patch antennas, (d)-(n) Comparison of active VSWRs and SLLs of synthesized array with those of uniformly spaced array.