# Simple Identification of Nonlinear Modal Parameters Using Wavelet Transform

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**ABSTRACT**

Estimation of the modal parameters of mechanical systems or structures is usually achieved by applying the well-known Frequency Response Function (FRF) method to experimental data obtained from free vibration after a shock excitation of the system or forced vibration using a variety of excitation signals. This method is however limited only to linear systems. The problem becomes more complex when nonlinear systems have to be identified. If the nonlinear system is 'well-behaved', i.e. if it shows periodic response to a periodic excitation, 'skeleton' identification using Hilbert Transform techniques may be used to estimate the modal parameters, in function of the amplitude and frequency of excitation. However, Hilbert transform has some limitations. This technique introduces more errors when high damping is present in the system. An improvement is offered by introducing Wavelet analysis to replace the Hilbert transform. This paper deals with the application of the aforementioned nonlinear identification techniques to an experimental mechanical system with backlash.

*Keywords: FRF, skeleton identification, Hilbert transform, Wavelet analysis*

## 1. Introduction

Estimation of modal parameters of linear mechanical structures is usually carried out by utilizing the Frequency Response Function (FRF) method using an experimental analysis such as free vibration with shock excitation or forced vibration with step or chirp excitation. However, there is a limitation that only linear dynamical systems can be tested through these methods. To overcome this difficulty, many researchers [[1],[2]] introduced nonlinear vibration system identification based on the Hilbert transform.

By definition [3], the Hilbert transform is a mathematical transform that shifts each frequency component of the instantaneous spectrum by $\pi/2$ without affecting the magnitude. Feldman [[4],[5]] has proposed methods of FreeVib and ForceVib to identify instantaneous modal parameters (natural frequencies, damping characteristics and their dependencies on a vibration amplitude and frequency). FreeVib is suitable for identifying the modal parameters of the system by free vibration analysis. However, when the system is well damped, ForceVib is more suitable.

These identification techniques prove to be very simple and effective, yet they have some limitations. Ruzzene et al. [6] show that the envelope and instantaneous frequency estimation in Hilbert transform technique introduce more errors when high damping is present in the system. An improvement can be achieved when Wavelet Transform is used instead of the Hilbert Transform to approximate the envelope signal and its instantaneous frequency. Wavelet Transform is a time-frequency representation (TFR) technique, which is developed as an alternative approach to the Short Time Fourier Transform to overcome the resolution problem. Compared to Wigner-Ville Distribution as another alternative of TFR, Wavelet Transform has more accurate modulus time-frequency components. Staszewski [7] extensively demonstrates this technique in his theoretical study. However, there are hardly any instances in the literature of the application of these techniques to real systems.

This paper presents such a practical application to a mechanical system with a backlash component. It compares and contrasts Hilbert transforms with Wavelet analysis in case of skeleton identification showing their possibilities and limitations.

## 2. Materials and Methods

A large number of signals, including vibration of nonlinear system can be converted to an analytic signal in complex-time and represented in the form of the combination of envelope and instantaneous phase [[1],[2]]:

$$Y(t) = y(t) + j\tilde{y}(t) = A(t) \cdot e^{j\varphi(t)} \qquad (2.1)$$

where $\tilde{y}(t)$ is the Hilbert Transform of the real-valued signal y(t), Y(t) is an analytic signal, A(t) and $\varphi(t)$ are an envelope (amplitude) signal and an instantaneous phase respectively. We now consider that the forced vibration equation of a single-degree-of-freedom system could be written as:

$$\ddot{y} + 2h_0(A)\dot{y} + \omega_0{}^2(A)y = F/m \qquad (2.2)$$

where y is the response signal, F is the excitation signal, m is the mass of the system, $h_0$ and $\omega_0$ are symmetrical viscous damping and stiffness characteristic of the system, respectively, which depend on the amplitude, A. According to the main properties of non-overlapping spectra of Hilbert Transform, Feldman [5] shows that equation (2.2) can be converted by Hilbert Transform to the complex form:

$$\ddot{Y} + 2h_0(A)\dot{Y} + \omega_0{}^2(A)Y = F/m \qquad (2.3)$$

where $Y(t) = A(t) \cdot e^{j\varphi(t)}$ is an analytic signal of a solution of the system and F(t) is the analytic signal of the forced excitation in complex-time form.
Substituting the analytic signal forms of Y(t) and F(t) together with the two derivatives of Y(t) in equation (2.3), the representation of the corresponding modal parameters can be derived [5]:

$$\omega_0^2(t) = \omega^2 + \frac{\alpha(t)}{m} - \frac{\beta(t)\dot{A}}{A\omega m} - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega} \qquad (2.4a)$$

$$h_0(t) = \frac{\beta(t)}{2\omega m} - \frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega} \qquad (2.4b)$$

where $\omega$ is the time derivative of the instantaneous phase $\varphi$, while $\alpha$ and $\beta$ are the real and imaginary parts of the ratio $\frac{F(t)}{Y(t)} = \alpha(t) + j\beta(t)$, respectively.

An improvement is proposed by introducing a Wavelet analysis for amplitude (A) and instantaneous frequency ($\omega$) estimation in equation (2.4). Wavelet analysis is done in a similar way to the Short Time Fourier Transform (STFT), in the sense that the signal is multiplied by a function (i.e. *mother wavelet*, similar to the window function in STFT), and the transform is computed separately for different segments of the time-domain signal.

## 3. Results and Discussion

An experiment was conducted on the second link of a two-link mechanism as shown in Figure 6 and Figure 7. The aim of this experiment is to identify the backlash size of the second link joint. For this purpose, certain degree of backlash (approximately $1.5^o$) was introduced in the joint of this link. The first link was kept fix while the second one was made to oscillate over a certain range. This link was driven by a servomotor through a toothed belt and a harmonic drive.
The vibration responses were measured with two rotary encoders. The first encoder measured the angular motion input to the harmonic drive, and the second one measured the relative oscillation between first link and second link. Therefore, the first encoder might be considered to measure excitation input of a *base motion system* in displacement form, while the second encoder measured the response of the base motion system.

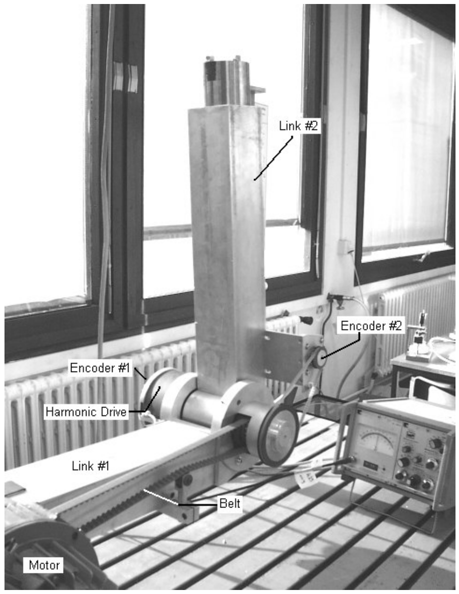


**Figure 6. Setup of two-link mechanism**

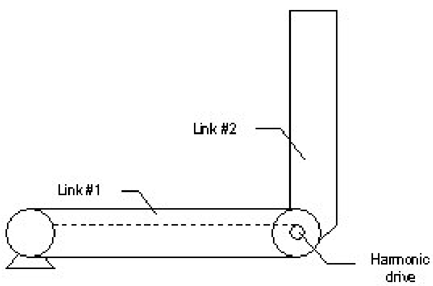


**Figure 7. Schematic drawing of a two-link mechanism**

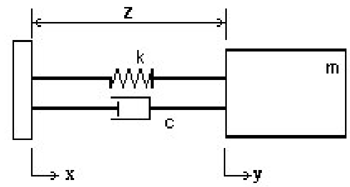


**Figure 8. Base Motion System**

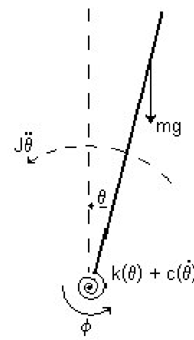


**Figure 9. Force balance diagram of link #2**

In the case of base motion system (Figure 8), where the excitation input is taken in the form of displacement, we need a little mathematical manipulation to solve the problem formulated in equation (2.2). The differential equation of displacement equation form for this problem can be written as:

$$\ddot{y} + 2h_0(A)\dot{y} + \omega_0{}^2(A)y = 2h_0(A)\dot{x} + \omega_0{}^2(A)x \qquad (3.1)$$

where: x = displacement input.

If we introduce z as a relative motion between x and y (z = y-x), we may rewrite equation (3.1) as:

$$\ddot{z} + 2h_0(A)\dot{z} + \omega_0{}^2(A)z = -\ddot{x} \qquad (3.2)$$

Taking $-\ddot{x}$ as input in equation (2.2), we can identify the modal parameters of the system.

The experiments were conducted by inputting linear chirp signal to the motor. The initial frequency of this signal is 0 Hz. The frequency continues to change at a constant rate, and it reaches 12 Hz in 30 sec. Relative motion between responses measured by both encoders, where in equation (3.2) was represented by z, can be seen in Figure 10. It is clearly seen from that figure, that the relative motion is higher than the backlash size in the joint. In the figure, the dashed lines represent the size of the backlash. At time 16.25 sec, which corresponds approximately to 6.5 Hz of excitation frequency, the relative motion of the link falls far below the size of backlash. This occurs due to insufficient energy of excitation. Such problem might cause unsatisfactory skeleton curve reconstruction, which is essential in modal parameter identification.

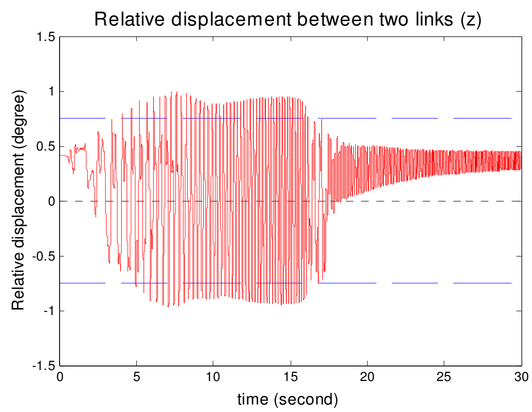


**Figure 10. Relative motion between both links (z)**

In order to check the validity of these identification techniques, the system has been modaly identified at the edge of backlash. The link was preloaded using a low stiffness spring to eliminate nonlinearity due to the backlash; hence the link was resting on one of the edges of the backlash. The natural frequency obtained is approximately 11 Hz, while from the mass line we can estimate the moment of inertia of the link to be approximately equal to 3.16 $kgm^2$.

Envelope and instantaneous frequency calculation of both input (x) and relative output signal (z) can be seen in Figure 11 and Figure 12, respectively. In the left side of both figures, we can see the envelope estimation of displacement input and displacement output. The light-dashed lines represent the envelope estimation based on Hilbert transform technique without any filtration. The filtered HT curves represent the same technique after low-pass filtration of 0.5% of its half sample rate (100% corresponds to half of the sample rate), while WT curves represent that of Wavelet Transform technique.

From these two figures, we can see that Wavelet analysis gives an improvement in estimation envelope and instantaneous frequency at several points. The most significant improvement is in the envelope and instantaneous frequency estimation of displacement output approximately after the first 17 seconds. If we refer back to Figure 10, it is clearly seen that the response is shifted after 17 seconds. This might have happened because at the corresponding time, the level of displacement input has fallen below the backlash size in the system. Hilbert Transform technique cannot estimate the envelope and also instantaneous frequency of a signal with certain offset. This is another advantage of Wavelet analysis.

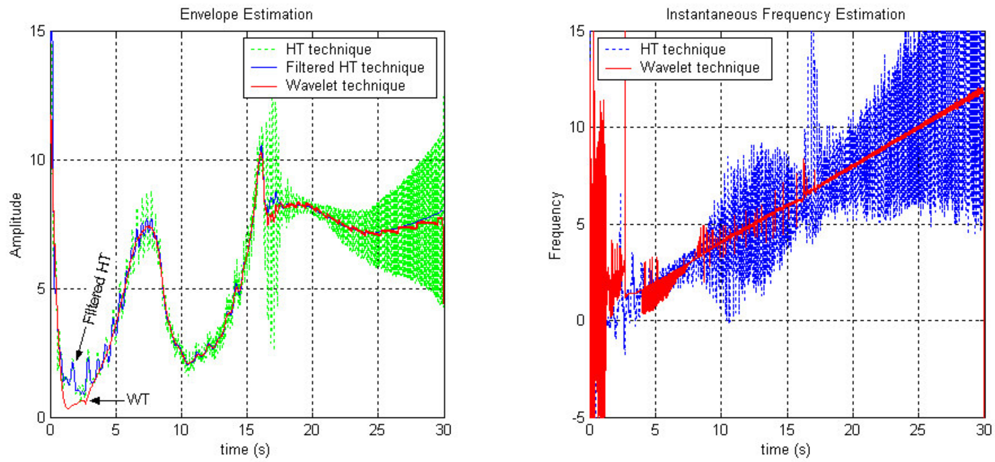


**Figure 11. Envelope and instantaneous frequency of input signal**

At approximately 16-17 seconds; we can see a significant estimation error in instantaneous frequency. At that instant, the estimation cannot be made in this region, since the frequency content of vibration response at that point corresponds to the resonance frequency of the system.

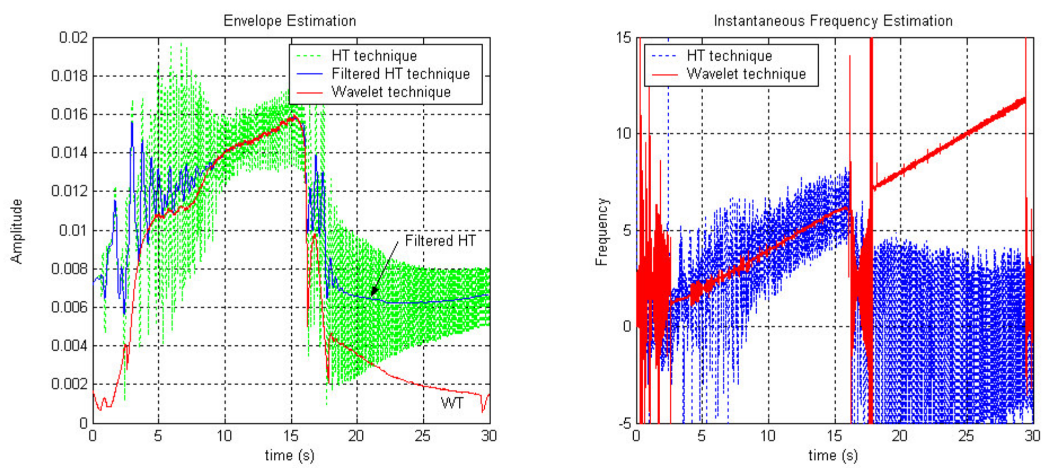


**Figure 12. Envelope and instantaneous frequency of output signal**

*Restoring Force*

The restoring force as function of displacement can be derived utilising equation (2.4a). The plot of this restoring force can be seen in Figure 13. In the left side of Figure 13, we can see the restoring force estimation based on Hilbert transform technique with low-pass filtration of 1% and 0.5% of half of its sample rate, respectively. In the right figure, we see the estimation based on Wavelet analysis. Approximately, the size of backlash obtained from both techniques is close to the real backlash size introduced in the system. From the figures, by applying regression on the reconstructed restoring force, we get the backlash size of 0.0258 rad (=1.48$^{o}$ with standard deviation of error below 1.5%), where the real backlash size is approximately 1.5° from manual measurement.

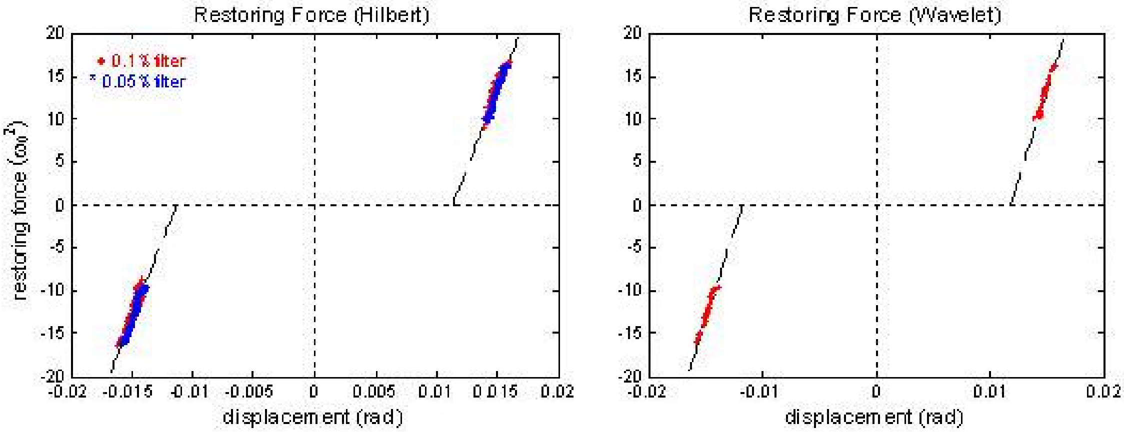


**Figure 13. Stiffness force estimation based on Hilbert and Wavelet transform**

Referring to equation (2.4a), the slope of the restoring force curve in Figure 13 actually represents the modal stiffness of the system. After multiplying the slope by moment of inertia of the second link, we can obtain the estimated stiffness value of the system. The angular stiffness parameter obtained by applying regression to the result is approximately 11000 Nm/rad.

The discontinuity appearing in the figures is due to the response in resonance region as mentioned before. Hence, we cannot obtain the stiffness force estimation in the corresponding region.

## 4. Conclusion

Nonlinear modal parameter estimation techniques based on Hilbert and Wavelet transform are shown to be a good approximation of the true modal parameters characteristics. Both the backlash size and the stiffness can be estimated satisfactorily.

An improvement of nonlinear modal parameters estimation by introducing Wavelet analysis has been achieved owing to several advantages offered by Wavelet properties. Wavelet transform is capable of analysing envelope and instantaneous frequency of a shifted signal and the results have less wiggles compared to those of Hilbert transform. However, there are some drawbacks in the Wavelet based technique. It needs not only a large amount of computation memory for its calculation process, since it deals with time-frequency domain, but also a long processing time.

A major difficulty in utilising the methods introduced in this paper concern the level of displacement input applied to the system. This displacement input should have an adequate level to ensure covering the backlash size. Due to the limitation of energy of excitation, this might become a problem, especially at high frequency. A limited level of excitation input will yield only a partial reconstruction of the restoring force characteristics.

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