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SLIDING MODE CONTROL FOR A FRICTIONAL SYSTEM

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Abstract – Friction is present in mechanical systems incorporating parts with relative motion. It is commonly found in position control systems, which normally contain various physical contacts in their drives or transmissions. In ideal linear systems, A PID feedback controller is adequate to provide accurate tracking. Unfortunately, friction introduces nonlinearity into the system, contributing significant influence on its performance. In such, friction has to be compensated in order to achieve accurate positioning. This paper presents position control of a mass sliding on the frictional surface using Sliding Mode Control compensation. Friction is generated using a generic friction model at asperity level, which incorporates friction behavior in both presliding and sliding regimes. This friction model mimics and generates virtual asperities in the two contacting surfaces. The friction compensation performance of the sliding mode controller scheme is compared and contrasted to the linear feedback and a combined feedback-feedforward control schemes. The latter scheme uses the inverse of the model in the feedforward block, while the friction force will be estimated using the most recent model of the Generalized Maxwell-Slip model. This friction model is a simplification of the generic friction model that makes it possible to be applied in the real time controlled system.

Keywords – sliding mode, robust control, friction, control

I. INTRODUCTION

Nonlinear system model imprecision may originate from the purposeful choice of a simplified representation of the system's dynamics. Modeling inaccuracies can be classified into two major types: structured (or parametric) uncertainties and unstructured uncertainties (or unmodeled dynamics) [1]. The first type corresponds to inaccuracies on the terms actually included in the model, while the second type corresponds to inaccuracies on the system order.

Modeling inaccuracies can have strong adverse effects on nonlinear control systems [2]. Robust control addresses issue relating to model uncertainty. A robust controller is composed of a nominal part, similar to a feedback control law, and an additional term aimed at dealing with model uncertainty. Sliding mode control is an example of a robust controller [3]. Consider a second order system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x) + g(x) \cdot u \qquad (1)$$

where $f(x)$ and $g(x)$ are nonlinear functions and $g(x)$ is a positive definite function.

The state x_1 is insured to be stable if,

$$\dot{x}_1 = -ax_1 \text{, for any } a > 0. \qquad (2)$$

Therefore, we can define a coordinate with respect to the corresponding stable manifold:

$$s = x_2 + ax_1 \qquad (3)$$
$$\dot{x}_1 = x_2 = -ax_1 + s \qquad (4)$$

Equation (3) is stable if s = 0. In order to evaluate the stability of this manifold, let us consider the Lyapunov candidate of $V = \frac{1}{2}s^2$. In the sense of Lyapunov stability, the derivative of V is necessary to be negative semi definite, i.e.

$$\dot{V} = \dot{s}s = s \cdot [\dot{x}_2 + a\dot{x}_1] = s \cdot [f(x) + g(x) \cdot u + ax_2] \leq 0 \qquad (5)$$

Introducing variable $\beta(x) = -[f(x) + ax_2]/g(x)$, stability of s is assured if:

$$u \begin{cases} < \beta(x) & \text{for } s > 0 \\ = \beta(x) & \text{for } s = 0 \\ > \beta(x) & \text{for } s < 0 \end{cases}$$

The above condition can be simplified using the control law:

$$u = \beta(x) - \kappa \cdot sign(s) \text{, where } \kappa > 0 \qquad (6)$$

A. Continuous Approximation of Switching Control Laws

The signum function control law u may cause an undesired chattering in the control input. This phenomenon can be avoided by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface. A boundary layer in the switching surface can be obtained by replacing the discontinuous jump in the signum function (6) of the control law by a line of finite slope as illustrated in Figure 1. The control input can then be formulated as follows:

$$u = \hat{u} - \kappa \cdot sat(s) \qquad (7)$$

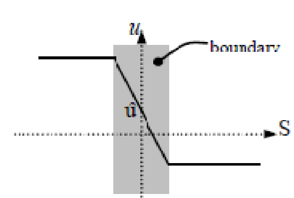

Figure 1. Interpolation of the Control Law

B. A Mass Sliding on the Frictional Surface

Friction behavior in mechanical systems is classically characterize using the Coloumb model. This model represents friction as a discontinuous signum function of velocity (v). Here, friction forces equal the static force value only at $v \neq 0$ and when $v = 0$ the characteristic simply sets the friction force below a certain value of the static force. In fact, at motion reversal ($v = 0$), friction shows a unique dependency on displacement. The region where this unique behavior exists is referred to as a presliding regime [4].

Thus, two different friction regimes exist: the *presliding regime*, where the friction force appears predominantly as a function of displacement in the small displacement range just after motion reversal; and the *sliding regime*, where the friction force is a function of sliding velocity. Friction behavior in presliding regime shows non-local memory hysteresis function to the displacement [5]. This behavior has two consequences: first, when the motion crosses zero velocity, instead of having discontinuous jump at zero velocity as in the Coloumb friction, the friction shows smooth transition from positive to negative static force and vice versa. However, this transition occurs in uncertain velocities around zero and is depending not only on the displacement but also the history of the motion (trajectory). Second, the friction-velocity relation at high velocity (sliding regime) cannot be simply represented by constant value of static force, yet it will have some uncertainties around the static force value. As an illustration, Figure 2 shows friction as a function of velocity under certain condition. The shaded-area encloses the friction behavior in presliding regime ranging from velocities of $-v_1$ to v_1.

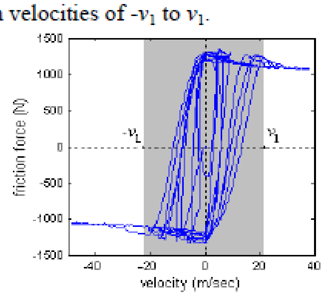

Figure 2. Plot of friction versus velocity at small displacement.

Friction behavior near zero velocity regions cannot be represented by the discontinuous jump as in the Coloumb friction model. At high velocity, where sliding motion exists, friction has some uncertainties. Friction force can be estimated with uncertainty band as a function of velocity and simplified using a discontinuous function of velocity $\hat{f}$. The estimation error, Δf can also be represented as a function of velocity. Figure 3 shows the proposed nominal friction function and the estimation error as a function of velocity. The actual friction function can be written as:

$$f(v) = \hat{f}(v) \pm \Delta f(v) \tag{8}$$

where $f(v)$ is the actual friction force, $\hat{f}(v)$ is the estimate of the nominal friction force and $\Delta f(v)$ is the estimation error of the friction force,

$$\hat{f}(v) = \begin{cases} F_s & ,|v| \geq v_1 \\ 0 & |v| < v_1 \end{cases} \tag{9}$$

where F_S is the approximation of static friction force, and v_1 represents the presliding regime uncertainty limit in velocity. The uncertainty band of the estimation error Δf as shown in Figure 3 (shaded-area) can be formulated as a discontinuous function of velocity as well and is represented as:

$$\Delta f(v) = \begin{cases} f_p & ,|v| \geq v_1 \\ f_s & |v| < v_1 \end{cases} \tag{10}$$

where f_P represents the maximum uncertainty for the sliding regime friction, and f_S for the pre-sliding regime. Figure 4 illustrates the uncertainty function.

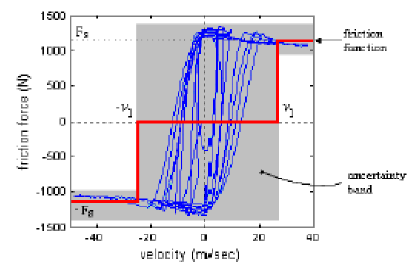

Figure 3. Friction approximation as a function of velocity and the uncertainty band with respect to the friction function. The bold-line represents the approximate friction function, while the gray-shaded is the uncertainty band.

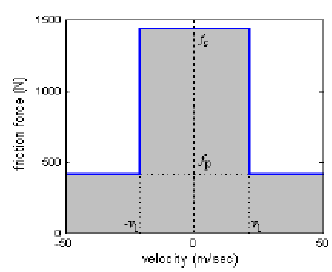

Figure 4. The uncertainty band as a function of velocity.

C. *Designing the Estimation Error Parameter, v_1*

Further analysis shows that the uncertainty limit of v_1 is not constant for any conditions. This uncertainty limit depends on the motion characteristic of the system, namely the acceleration and the frequency content of the trajectory. A higher frequency content of the reference trajectory will results in higher v_1 values in the uncertainty band.

However, for a designated reference trajectory, the frequency content can be calculated and the uncertainty parameter v_1 can also be predicted. In order to validate the influence of the characteristics of the reference signal on the uncertainty parameter v_1, a pure sinusoidal signal is subjected to the friction model. The evolution of the parameter v_1 as the property of the reference signal changes will be observed. Figure 5 shows the results of this observation.

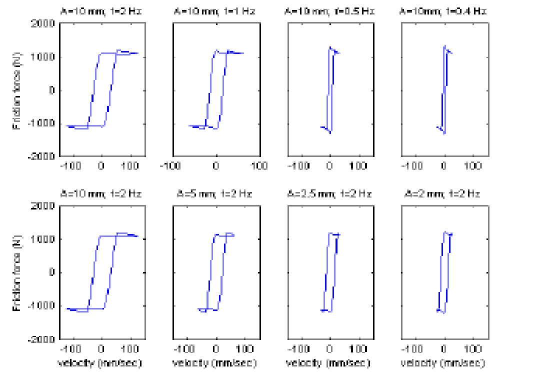

Figure 5. Plot of friction force versus velocity for varies reference sinusoidal signal showing the evolution presliding regime band v_1. Amplitude and frequency of the reference signal are notated by A and f, respectively.

The top panels of Figure 5 show the evolution of the uncertainty band parameter v_1, when the frequency of the reference sinusoidal signal is decreasing from left to right, while the bottom panels show the evolution of the same parameter when the amplitude is changing. The uncertainty band parameter v_1 decreases as the frequency of the reference sinusoidal signal decrease. Similar behavior is also observed as the amplitude of the reference signal is decreased. However, in respect of the frequency of the reference signal, the parameter v_1 evolves at different rate compared to that in the amplitude of the reference signal. Parameter v_1 evolves linearly proportional to the reference signal frequency, while this parameter also appears to be a square root function in the reference signal amplitude, as shown in Figure 6. These relationships are used in order to design the uncertainty bound function.

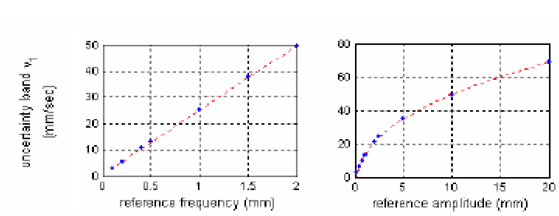

Figure 6. Plot of the uncertainty band v1 versus amplitude and frequency of the reference sinusoidal signal.

D. Application of the SMC in Tracking Control Problem of a Frictional System

In this paper, friction is compensated using the SMC approach utilizing the modeled phenomena of friction such as the presliding and sliding regime limits. In literature, various controller techniques have been demonstrated to compensate friction [6]. Among these controllers, is the backstepping technique **Error! Reference source not found.**. However, this controller only approximates the friction model with a signum function similar to the Coulomb model and does not consider the presliding information. As an alternative to friction compensation, feedforward controller utilizing the friction model in combination with a PID controller is shown to give satisfactory results [7]. However, feedforward compensation is not robust against system changes.

Consider a mass sliding on the frictional surface. The equation of motion is written as:

$$m\ddot{x} + f(x,\dot{x}) = u \tag{11}$$

where u is the control input, x is the (scalar) output of interest, and the dynamics of friction $f(x,\dot{x})$ is not exactly known, but estimated as $\hat{f}$ (eq. 9). The estimation error on f is assumed to be bounded by some known function Δf as shown in eq. (10):

$$|\hat{f} - f| \le \Delta f \tag{12}$$

In order to control the tracking trajectory of the system, $x(t) \equiv x_d(t)$, a first order sliding surface $s = 0$, is defined, namely:

$$s = \left(\frac{d}{dt} + \lambda\right)\tilde{x} = \dot{\tilde{x}} + \lambda\tilde{x} \tag{13}$$

where $\tilde{x} = x - x_d$.

For stability condition based on Lyapunov stability criterion, the derivative of s in time has to be zero:

$$\dot{s} = \ddot{\tilde{x}} + \lambda\,\dot{\tilde{x}} = \ddot{x} - \ddot{x}_d + \lambda\,\dot{\tilde{x}} \tag{14}$$

Referring to eq. (9),

$$\dot{s} = f + u - \ddot{x}_d + \lambda\,\dot{\tilde{x}} = 0 \tag{15}$$

The best approximation for $\hat{u}$ is thus:

$$\hat{u} = -\hat{f} + \ddot{x}_d - \lambda\dot{\tilde{x}} \tag{16}$$

In order to insure the stability in the sliding surface, we add to $\hat{u}$ a term discontinuous across the surface $s = 0$,

$$u = \hat{u} - \kappa \cdot sign(s) \tag{17}$$

where *sign* is the signum function.

By choosing κ to be large enough, we can guarantee that the stability condition is always verified. This can be assured by selecting:

$$\kappa = \Delta f + \eta \tag{18}$$

where η is any positive number.

Therefore, we can represent the control input u as:

$$u = \ddot{x}_d - \hat{f}(\dot{x}) - \lambda\dot{\tilde{x}} - \kappa \cdot sign(s)$$

$$u = \ddot{x}_d - \begin{Bmatrix} F_s & ,|\dot{x}| \ge v_1 \\ 0 & |\dot{x}| < v_1 \end{Bmatrix} - \lambda\dot{\tilde{x}} - \begin{Bmatrix} f_p + \eta & ,|\dot{x}| \ge v_1 \\ f_s + \eta & |\dot{x}| < v_1 \end{Bmatrix} \cdot sign(\dot{\tilde{x}} + \lambda\tilde{x}) \tag{19}$$

II. SIMULATION RESULTS

Designated reference signal, namely a filtered random signal with certain frequency content is subjected to the system for simulation purposes. The frequency content of this reference signal does not exceed the fix uncertainty band v_1. The filter used in this signal was designed with a 4^{th} order digital Butterworth filter and 2 Hz cutoff frequency. The corresponding reference signal will be used to validate the compensation scheme using the SMC. The results are compared to other different compensation schemes, namely P-Controller and P + Feedforward. The P-Controller uses feedback compensation with proportional controller, while the P + Feedforward uses a combined feedback and feedforward compensation. The feedforward block in the latter scheme simply consists of the inverse of the system, which contains the inertia force and the friction force estimation. The most recent model of friction, namely *the Generalized Maxwell-Slip* [9], is utilized in the feedforward control to estimate the friction force in the system, while the inertia force is estimated by using the second derivative of the reference signal and multiply it by the mass in the system. The gains in both P-Controller and P+Feedforward controller have been optimized for minimum error. Figure 7 shows the reference signal with tracking error of each friction compensation scheme, while Table 2 shows the quantification value of the corresponding racking errors in the RMS and the maximum error value.

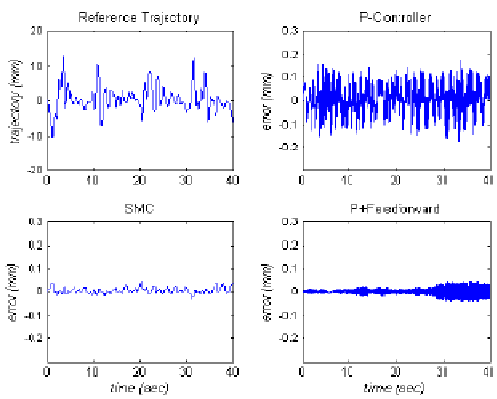

Figure 7. Error compensation result of a frictional system for different compensation schemes. Top-left plot shows the reference trajectory of the system. The top-right figure depicts the error for the system with proportional controller. The bottom-left shows the error for the system with SMC, while the bottom-right shows the result for a system with combined feedback-feedforward compensation.

Table 2. Performance of friction compensation schemes.

Compensation	*A. RMS(mm)*	*max. error(mm)*
P-controller	0.0610	0.1705
P+Feedforward	0.0144	0.0425
SMC	0.0138	0.0404

Friction compensation using P-controller yields the largest error. This can be understood since the system incorporates strong nonlinearity of friction and P-controller deals only with linear system. However, the performance of P+Feedforward and the SMC compensation, result in almost identical performances. Based on the performance values, the SMC offers slightly better result compared to the P+Feedforward compensation.

In the second simulation, a noise disturbance signal is introduced to the control input. The disturbance is characterized by the normal Gaussian distribution with approximately 5% of the average reference signal. Figure 8 shows the tracking error of the system while Table 3 depicts the performance of each compensation scheme. These results highlight a significant advantage of SMC. The P plus feedforward scheme is obviously very sensitive to disturbance in the system; on the other hand, the result of the SMC scheme is not significantly influenced. The performance of the former compensation scheme is reduced by a factor of five when 5% white noise is perturbed to the system. In contrast, the performance drop of the SMC is negligible.

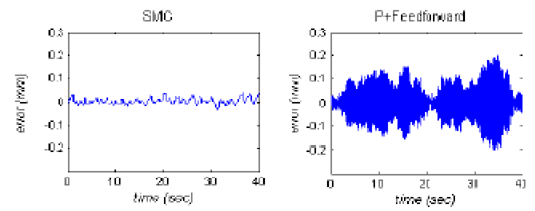

Figure 8. Error compensation result of a frictional system with 5% noise disturbance input.

Table 3. Performance of friction compensation schemes

Compensation	*B. RMS(mm)*	*max. error(mm)*
P+Feedforward	0.0665	0.2009
SMC	0.0138	0.0402

Interpolating the control input of the above SMC experiment in a thin boundary layer is expected to reduce the high control chattering. A boundary layer thickness of 0.02 mm results in low chattering with minimum error. Figure 9 shows the result of the corresponding continuous approximations of switching control laws. The top panels show the tracking error with the control input of the SMC system before the continuous approximation applied. Note that the tracking error of this plot is similar to that in Figure 7.

The bottom panels show the tracking error and the control input for the SMC system with continuous approximation of switching control laws. This improvement gives performance RMS values of 0.0295 mm and 0.0753 mm of maximum error. Compared to the result obtained before introducing the boundary layer, the performance of this latter model is not as good as the former. However, this result is still acceptable. Control chattering reduces tracking performance.

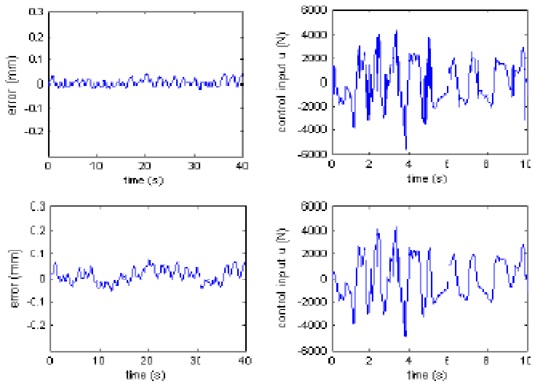

Figure 9. Switch and smooth tracking performance and control input. The top panels show the tracking performance and control input for the switch control system, while the bottom panels for the smooth control system.

III. CONCLUSION AND DISCUSSION

The SMC scheme offers a robust control compared to the feedforward scheme, even in some cases, for an ideal system with low disturbance, feedforward scheme gives very good performance. Adaptive SMC might be possible to be applied, in the sense that the uncertainty band (in particular the parameter v_1) is self-adjusted based on the amplitude and the frequency content of the reference signal.

For real practical application, friction behavior should be identified in advance in order to find the controller variables F_S, f_S, f_P and v_1. Some identification techniques for real application are presented and studied (0)

REFERENCES

[1]. Worden, K. and Tomlinson, G.R., (2001), Nonlinearity in Structural Dynamics, Detection, Identification and Modelling, Institute of Physics Publishing

[2]. Schoukens, J. and Pintelon, R., (1991), Identification of Linear Systems, A Practical Guideline to Accurate Modeling, Pergamon Press

[3]. Kwatny, H.G. & Blankenship, G.L., (2000). Nonlinear Control and Analytic Mechanics, Birkhäuser, Boston.

[4]. Prajogo, T. (1999), Experimental Study of Pre-rolling Friction for Motion-Reversal Error Compensation on Machine Tool Drive Systems, PhD thesis, Department Werktuigkunde, Katholieke Universiteit Leuven, Belgium.

[5]. Al-Bender, F., V. Lampaert & Swevers, J. (2004), 'Modeling of Dry Sliding Friction Dynamics: From Heuristic Models to Physically Motivated Models and Back', Chaos: An Interdisciplinary Journal of Nonlinear Science, 14(2), 446-460.

[6]. Jamaludin, Z., Van Brussel, H., Swevers, J., (2009), Friction Compensation of a XY Feed Table using Friction Model-Based Feedforward and an Inverse-Model-Based Disturbance Observer, *IEEE Trans. on Industrial Electronics- Advances in Motion Control*, VOL. 56, NO. 10.

[7]. Tjahjowidodo, T., Al-Bender, F. & Van Brussel, H. (2005). Friction Identification and Compensation in a DC Motor, in 'Proceedings of 16th IFAC World Congress', Prague, Czech Republic, Paper ID: 4017.

[8]. Vo Minh, T., Tjahjowidodo, T., Ramon, H., and Van Brussel, H., (2010). Cascade Position Control of a Single Pneumatic Artificial Muscle-Mass System with Hysteresis Compensation, accepted to appear IFAC Mechatronics Journal.

[9]. Lampaert, V., Al-Bender, F. & Swevers, J. (2003b), A Generalized Maxwell-Slip Friction Model Appropriate for Control Purposes, in 'Proceedings of the 2003 International Conference on Physics and Control', Saint-Petersbourg, Russia, pp. 1170-1178

[10] Tjahjowidodo, T. (2006), Characterization, Modelling and Control of Mechanical Systems Comprising Material and Geometrical Nonlinearities, PhD thesis, Department Werktuigkunde, Katholieke Universiteit Leuven, Belgium.