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Integrated Rate and Inertial Dependent Prandtl-Ishlinskii Model for Piezoelectric Actuator

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Abstract— **Piezoelectric drive mechanisms consist of piezoelectric materials that actuate the mechanism at different ranges of frequency, as the piezoelectric actuators are subjected to a nonlinear phenomenon of hysteresis which is sensitive to frequency changes. Designing an adaptive hysteretic model facilitates the operation of such mechanisms in general control framework. In this paper, an Integrated Rate and Inertial dependent Prandtl-Ishlinskii model using an exponential function for varying damping factor is proposed for positioning control of a piezoelectric actuator at frequency range between 1 Hz to 200 Hz.**

Keywords-piezoelectric actuator; Prandtl-Ishlinskii Model; Hysteresis Loop; Rate-Dependent Model; Inertial-Dependent Model

I. INTRODUCTION

In recent years, applications of piezoelectric actuators in piezoelectric motors and mechanisms have been widely explored [1-6]. The apparent characteristics of the piezoelectric actuators such as fast frequency response and high resolution down to nanometer level have encouraged researchers as well as industries to develop micro-positioning devices. In spite of the advantage for micro-positioning application, the piezoelectric actuators possess severe nonlinearity due to a hysteresis relation between the voltage input and the displacement output. This phenomenon degrades the performance of the actuator and the mechanism in positioning control applications as it causes inaccuracy in the open loop system, inadvertent oscillations of the system and even instability of a closed loop system [7, 8]. Moreover, the hysteresis phenomenon in piezoelectric actuators is dependent on the rate of applied input and aging of the actuators [9]. In other words, the intensity of hysteresis increases with the increase in operating frequency and vice versa. As a result, developing a frequency-dependent or rate-dependent model of hysteresis has attracted significant attentions of researchers. Considering the works done in this area, readers may refer to Al Janaideh [10], who proposed a rate-dependent Prandtl-Ishlinskii model by presenting a threshold value and density function, which appear as a function of varying input rate to characterize the hysteresis in the piezoelectric actuator at frequency range between 0.1 Hz to 200 Hz. A different approach was proposed by Ang [11] to model the hysteresis at low frequencies up to 50 Hz utilized a linear function relating the slope of the hysteresis loop to input rate applied to the actuator. Application of different periodical inputs motivated Al Janaideh [12] to consider an initial rate-dependent model at higher frequency range between 0.1 Hz to 500 Hz. The relation between threshold value and the rate of input applied to piezoelectric actuator was considered to gain a better approach to develop rate-dependent Prandtl-Ishlinskii model [13]. The inverse rate-dependent model was proposed by Al Janaideh [14] to be used as a feed forward controller in a control loop for positioning control of an actuator. Since the hysteresis loops sometimes also exhibit asymmetrical shape, a generalized rate-dependent hysteresis model was proposed further to capture the asymmetric shape of the hysteresis in a rate-dependent Prandtl-Ishlinskii model [15]. Ang [16] designed a feedforward controller through utilization of inverse of the rate-dependent model that formerly presented in [11].

In this paper, an inertial-dependent Prandtl-Ishlinskii model is proposed in term of the stop operator which is one of the two essential yet well-known operators of the Prandtl-Ishlinskii model. Using an exponential function that expresses relation between damping factor and the changes in frequency directs us to convert the initial rate-independent model into a rate-dependent model of the hysteresis at frequency range between 1 Hz to 200 Hz for the applied input voltages up to 80V.

II. CLASSICAL PRANDTL-ISHLINSKII MODEL

This model exploits two well-known operators i.e., a play operator and a stop operator. The stop operator plays as an inverse of the play operator and can be solely used in a feed forward controller scheme to mitigate the hysteretic effects in piezoelectric actuators. The properties of these operators are discussed in this section.

A. Stop Operator

Suppose $C_m[0,t_E]$ is a space of piecewise monotone continuous functions. For any input $v(t)\in C_m[0,t_E]$, the stop operator is defined by the following expressions,

$$e_r(v)=\min(r,\max(-r,v)) \quad (1)$$

$$E_r[v;w_{-1}](0)=e_r(v(0)-w_{-1}) \quad (2)$$

$$E_r[v;w_{-1}](t) = e_r(v(t) - v(t_i) + E_r[v;w_{-1}](t_i)) \quad (3)$$

$$t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1$$

where, $0 = t_0 < t_1 < \ldots < t_N = t_E$, $v(t)$ is the input value, $w(t)$ is the output and r is the threshold value. Finally, the output of the stop operator will be obtained using the following numerical expression,

$$w(t) = \sum_{i=1}^{k} P(r_i) E_{r_i}[v] \Delta r \quad (4)$$

where, i is the number of samples and $P(r_i)$ is the density function which is applied to adjust the slope of the hysteresis loop [8, 17].

B. Play Operator

Similarly, for the play operator, let us consider a space of piecewise monotone continuous function which is defined by $C_m[0, t_E]$. For any input $v(t) \in C_m[0, t_E]$, the play operator is expressed as follows,

$$f_r(v, w) = \max(v - r, \min(v + r, w)) \quad (5)$$

$$F_r[v; w_{-1}](0) = f_r(v(0), w_{-1}) \quad (6)$$

$$F_r[v; w_{-1}](t) = f_r(v(t), F_r[v; w_{-1}](t_i)) \quad (7)$$

$$t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1$$

where, $0 = t_0 < t_1 < \ldots < t_N = t_E$, $v(t)$ is the input value, $w(t)$ and $w_{-1}(t)$ are subsequently the output value and initial value of the output and r is the threshold value. Finally, the output of the play operator will be obtained using the following expression,

$$w(t) = P_0 v(t) + \sum_{i=1}^{k} P(r_i) F_{r_i}[v] \Delta r \quad (8)$$

where P_0 and $P(r)$ are subsequently the initial value and the subsequent value of the density function which are used to adjust the slope of the hysteresis loop with the actual hysteresis loop [8, 17].

III. Inertial-Dependent Prandtl-Ishlinskii Model

As the presence of nonlinear phenomenon of hysteresis is inevitable in the piezoelectric actuators, the linear constitutive equation determined to describe the dynamic characteristics of the piezoelectric actuator [18] cannot be solely applied to capture the nonlinearity of hysteresis existing in these types of materials.

Additionally, the existing models of the hysteresis such as Prandtl-Ishlinskii model have some restrictions operationally to characterize the hysteresis appropriately. These restrictions are attributed to the inconsistency of this model in contact with high frequency operation and inertial effect of the actuator, which is not negligible at high frequency as well as high input magnitude applied to the actuator.

In order to resolve this problem, let us assume the actuator as a mass-spring-damper system and incorporate the model with the classical Prandtl-Ishlinskii model which can improve the modeling of the hysteresis drastically. The so-called inertial-dependent Prandtl-Ishlinskii model is expressed in terms of the play and the stop operators as follows:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x - F_{preloading} = f_{playoperator}(V) \quad (9)$$

$$f_{stopoperator}(m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x - F_{preloading}) = V \quad (10)$$

where, m_{eq}, c_{eq}, k_{eq} are the equivalent mass, damping and stiffness factors, $F_{preloading}$ is the preloading force applied to the piezoelectric actuator and V is the applied voltage. Incorporating these terms in the Prandtl-Ishlinskii model converts this model into the inertial-dependent model due to the term of $m_{eq}\ddot{x}$. As can be observed, the first and the second terms between bracket in (10) incorporate the $\ddot{x}$ and $\dot{x}$ terms. These multipliers are also the function of frequency due to transformation that can be made for these terms in sense of frequency responses of $\ddot{x} = \omega^2 x$ and $\dot{x} = \omega x$ in which the ω represents the frequency. To show the performance of the inertial-dependent model, Fig.1 and Fig.2 compare the output of the classical Prandtl-Ishlinskii model and output of the inertial-dependent model for piezoelectric actuator under excitation at input frequency of 200 Hz with 40V magnitude.

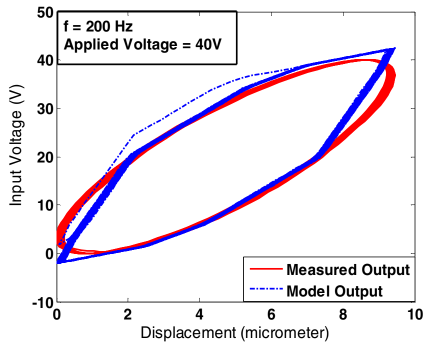

Figure 1. Modeling with classical Prandtl-Ishlinskii model

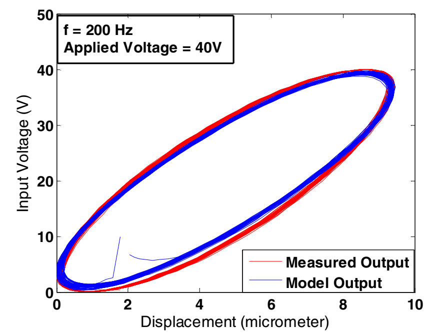

Figure 2. Modeling with inertial-dependent Prandtl-Ishlinskii model

It can be concluded that the quality of characterization is improved when the inertial and damping terms are taken into consideration in the Prandtl-Ishlinskii model.

IV. Integrated Rate and Inertial Dependent Prandtl-Ishlinskii Model

The variations of parameters in the inertial-dependent model reveal that some of the parameters are varying with frequency. One of these parameters, which exhibit significant change, is the equivalent damping factor, c_{eq} . As illustrated in Fig.3, this parameter varies exponentially at frequency interval between 10 Hz to 200 Hz for two sets of sinusoidal input voltages with magnitudes of 40V and 80V.

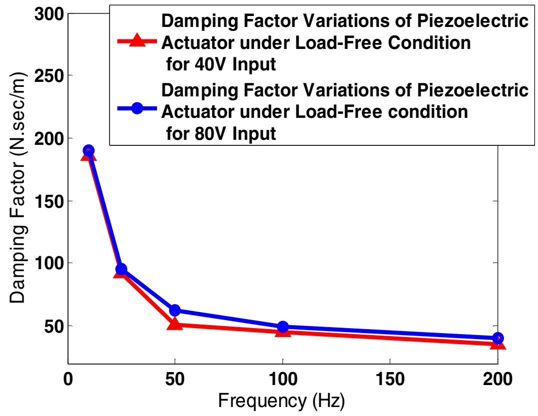

Figure 3. Rate of change of prameter c_{eq} versus frequency between 10 Hz to 200 Hz for two sets of sinusoidal inputs with amplitudes of 40V and 80V.

In order to improve the model to capture the characteristics in wide range of operating frequency, a rate-dependent inertial model based on the variations of the equivalent damping factor is proposed. Following this idea, variation of the damping factor is approximated by using an exponential function as a function of frequency. The proposed exponential function is written as

$$C_{eq} = q_1 \exp(-f) + q_2 \qquad (11)$$

where, q_1 and q_2 are constant values and f represents the amount of frequency for a predetermined range between 10 Hz to 200 Hz. The constants q_1 and q_2 will be obtained through parameter estimation, which will be performed in MATLAB optimization toolbox.

V. Experimental Set-up

In order to carry out an experiment on the piezoelectric actuator, a dedicated set-up was prepared. In the experiment, a piezoelectric actuator (P-887.90 from PHYSIK Instrument Co.) is used as main element. The chosen actuator is able to generate 36±10% µm stroke at maximum applied voltage of 120V. The signal sent to the actuator is amplified by gain 10 using a suitable amplifier (E.663, PHYSIK Instrument Co.) designed for this type of actuator. A capacitive sensor (CS1, Micro-Epsilon Co. with resolution of 0.75nm and operating temperature range of -50°C to 200°C) is employed to capture the displacement of the actuator with high degree of accuracy. A schematic diagram of the experimental set-up is illustrated in Fig.4.

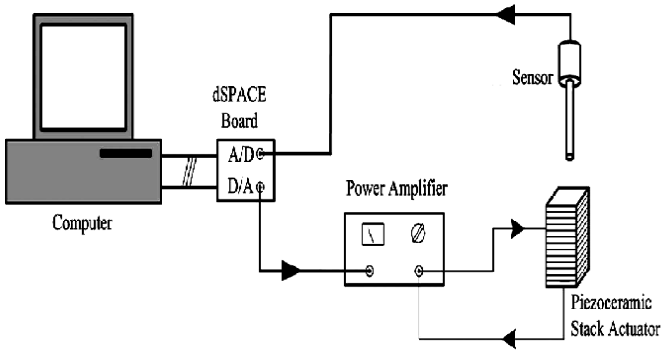

Figure 4. Set-up component

VI. Modeling Results

In this section, the dedicated set-up was operated through the dSPACE acquisition system. Two sets of sinusoidal inputs with amplitudes of 40V (20V±20V bias) and 80V (40V±40V bias) were applied to the piezoelectric actuator at different frequencies of 10 Hz, 25 Hz, 50 Hz, 100 Hz and 200 Hz.

Four stop operators were utilized to capture the hysteresis in the form of rate and inertial dependent Prandtl-Ishlinskii model. As the stop operator plays as an inverse of the play operator, this allows us to implement it in a feedforward control scheme to mitigate the effect of hysteresis nonlinearity .

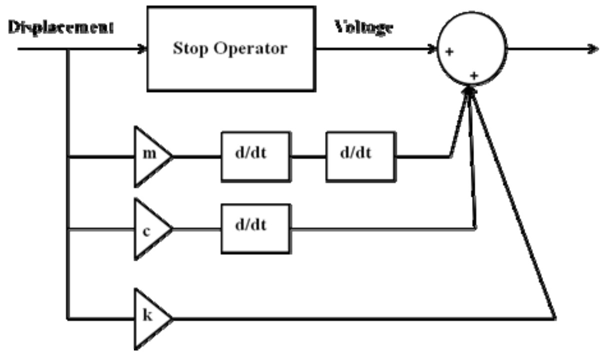

Figure 5. Mechanism for inertial-dependent stop operator

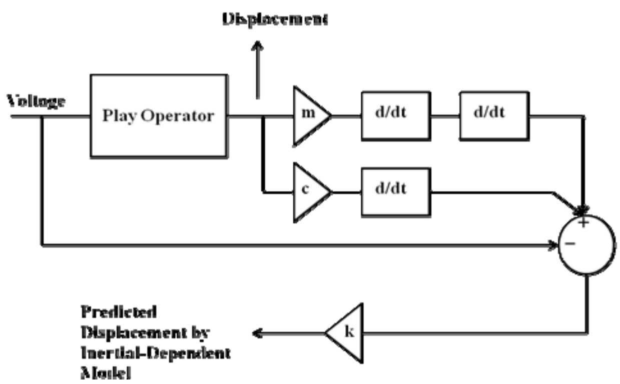

Figure 6. Mechanism for inertial-dependent play operator

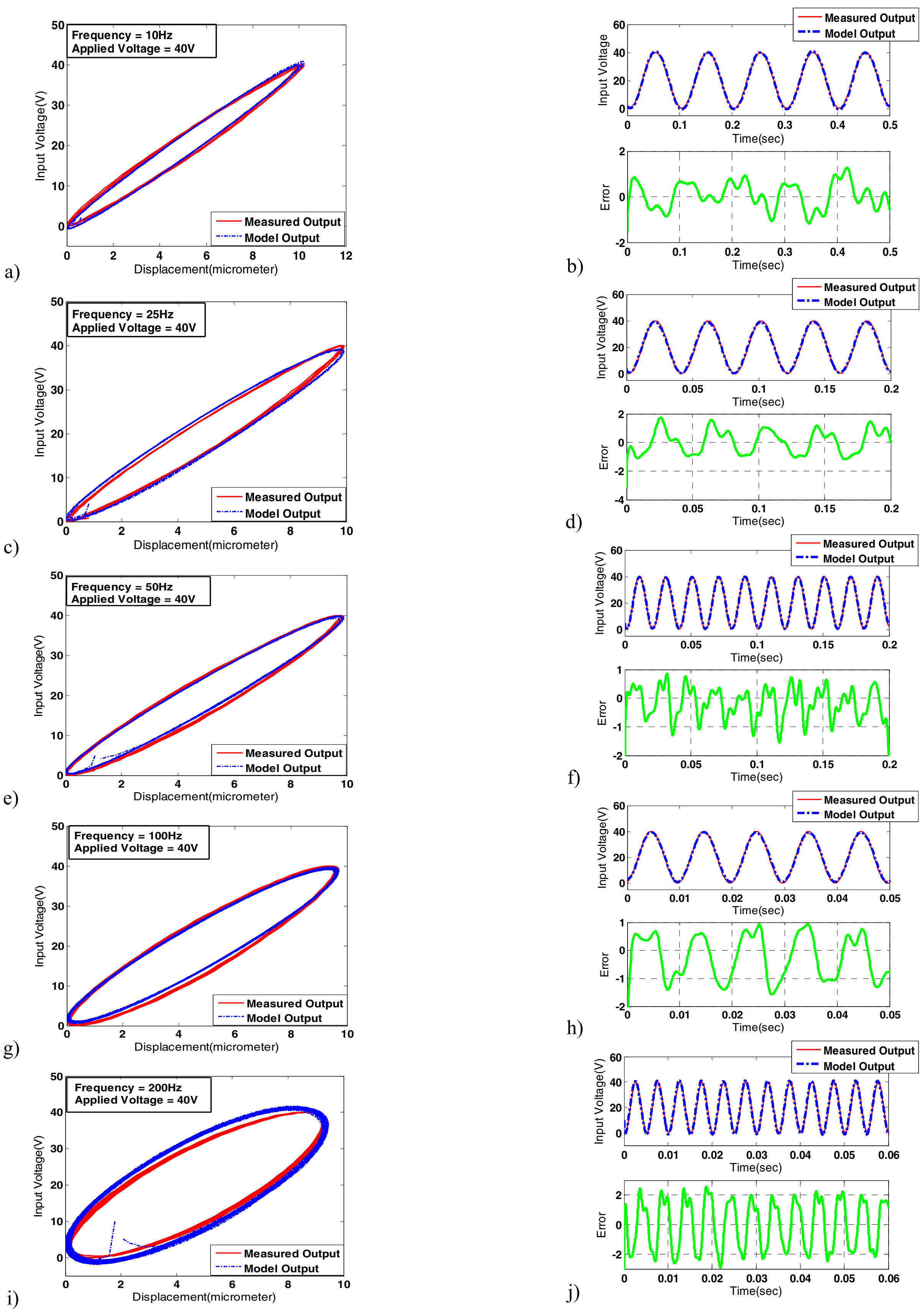

Figure 7. a&b show the results at frequency of 10 Hz; c&d: 25 HZ; e&f: 50 Hz; g&h: 100 Hz; i&j:200 Hz.

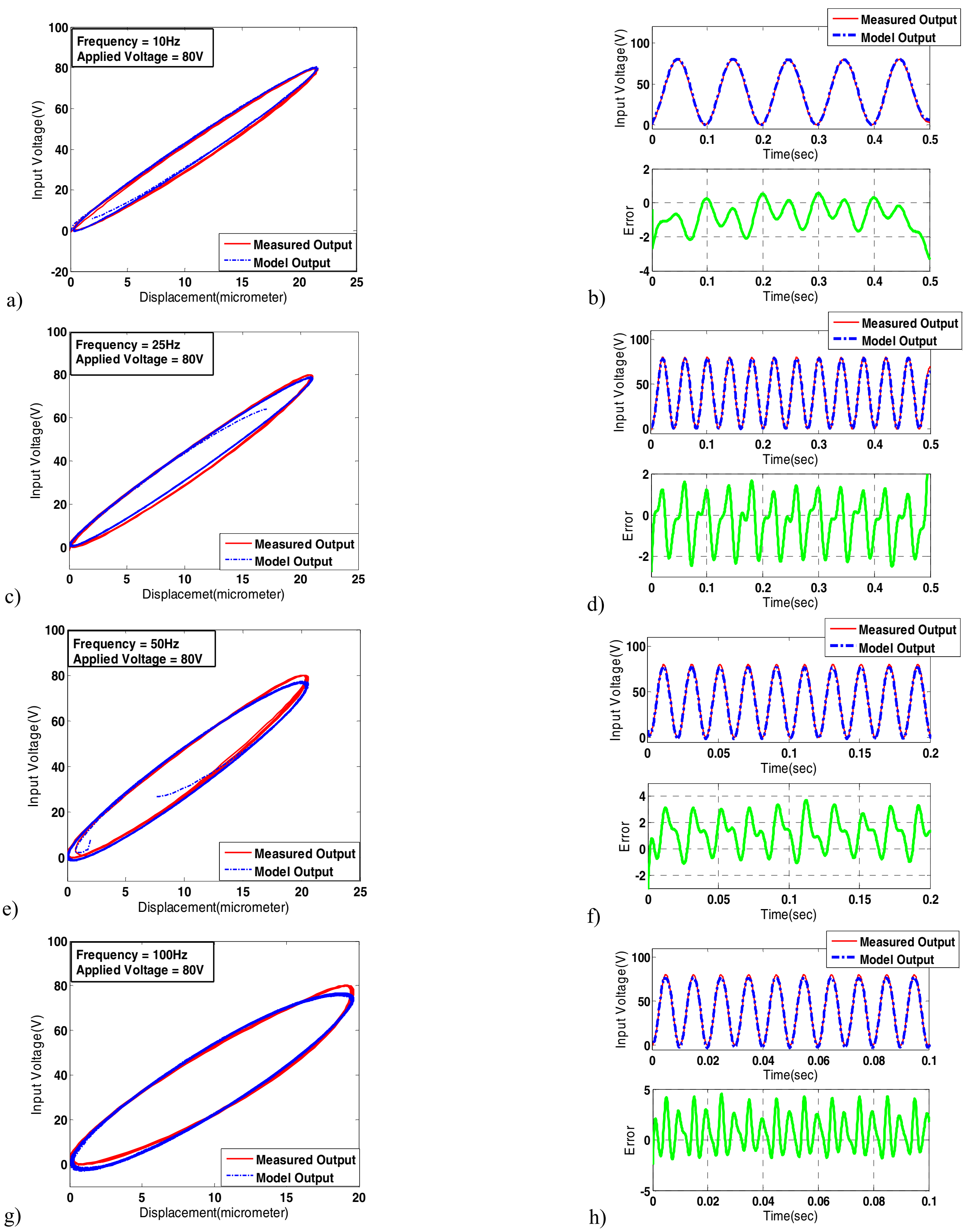

Figure 8. a&b show the results at frequency of 10 Hz; c&d: 25 HZ; e&f: 50 Hz; g&h: 100 Hz;

The two terms $\dot{x}$ and $\ddot{x}$ in the inertial dependent model are obtained by differentiating the input displacement. The ability to implement the derivative of input displacement in the stop operator can also be applied to differentiate the output displacement of the play operator. Unlike the stop operator, it will cause a numerical problem due to the discontinuity at turning points of the hysteresis loops in the play operator. The mechanisms of the inertial-dependent model are displayed in Fig.5 and Fig.6 in which *m, c, k* are subsequently equivalent mass, damping and stiffness of the piezoelectric actuator that has to be identified in the parameter estimation process.

Applying (11) in the inertial-dependent model leads to the results as shown in Fig.7 and Fig.8 for both types of low magnitude input of 40V as well as high magnitude input of 80V. The results of the modeling with rate and inertial dependent model for 40V sinusoidal input set are shown in Fig.7, while the results of the modeling for 80V sinusoidal input set at different frequencies are shown in Fig.8.

The results show that this model can be applied as an adaptive feed forward control scheme to enable the system to operate at a wide-band frequency.

The degree of accuracy is dependent on the optimization method utilized for parameter estimation particularly for q_1 and q_2 which determines the damping factor variations versus frequency. The values obtained for the low and high amplitude sets of inputs are subsequently 2.8×10^6 for q_1 and 55 for q_2.

VII. CONCLUSION

In this study, an inertial-dependent Prandtl-Ishlinskii model was proposed to overcome the substantial difficulties in the classical Prandtl-Ishlinskii model to capture the hysteresis behavior especially at high frequency range. The variation of various parameters was considered in the inertial-dependent model and it was observed that variation of the equivalent damping factor versus frequency was more prominent compared to the other parameters. This variation was mathematically modeled using an exponential function, while the constants of this function were estimated along with other parameters in the integrated rate and inertial dependent Prandtl-Ishlinskii model.

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