

## **THERMAL INTERACTION BETWEEN FREE CONVECTION AND FORCED CONVECTION ALONG A CONDUCTING PLATE EMBEDDED IN A POROUS MEDIUM**

***J.-J. Shu***

*Department of Pharmacy, National University of Singapore, Singapore*

***I. Pop***

*Faculty of Mathematics, University of Cluj, Cluj, Romania*

*The article investigates the conjugate problem of free convection and forced convection along a conducting vertical plate separating two semi-infinite porous reservoirs maintained at different temperatures. The mean heat flux through the plate and the wall temperature distribution on the both sides of the plate are determined. The results show the effects of the resistance parameter  $R_p$  and free-to-forced-convection parameter  $R_f^*$  on the mean heat transfer through the plate and on the wall temperatures of the both sides of the plate.*

### **INTRODUCTION**

The problem of free convection due to a heated vertical plate of finite extent placed adjacent to a semi-infinite porous medium is of considerable theoretical and practical interest. To a large extent, this interest has been motivated by such diverse engineering problems as geothermal energy extraction, energy conservation in buildings, thermal insulations, packed-bed reactors, sensible heat storage beds, ceramic processing, and groundwater pollution. An excellent description of the state of the art of this subject has been summarized in recent books by Nield and Bejan [1] and by Nakayama [2].

In recent years, in order to take account of physical reality, there has been a tendency to move away from considering idealized problems in which the plate is considered to be infinitesimally thin, but instead to take account of the so-called conjugate effects which arise due to the finite thickness of the plate. This case has been studied theoretically by several investigators [3-8].

In the present article, we investigate the conjugate problem of free convection along one side of a vertical thin plate and a forced-convection flow on the other side of the plate, which is embedded in a porous medium, with consideration of the plate thermal resistance. The transformed boundary-layer equations are solved numerically following the same finite-difference method as that recently used by the present authors [9]. The numerical results for the mean Nusselt number and wall temperatures on both sides of the plate have been obtained and discussed in detail.

It is worth mentioning to this end that Bejan and Anderson [10] presented the first analytical treatment of the problem of counterflow on both sides of a vertical plate separating two semi-infinite porous reservoirs maintained at different temperatures. How-

## NOMENCLATURE

<p><math>b</math> thickness of the plate</p> <p><math>f</math> reduced stream function, Eq. (10)</p> <p><math>g</math> acceleration due to gravity</p> <p><math>G</math> operator</p> <p><math>k</math> thermal conductivity of the porous medium</p> <p><math>k_s</math> thermal conductivity of the solid plate</p> <p><math>K</math> permeability of the porous medium</p> <p><math>L</math> length of the plate</p> <p><math>Pe</math> Peclet number, Eq. (11)</p> <p><math>q_{s,c}</math> local heat flux, Eq. (8)</p> <p><math>Ra</math> Rayleigh number, Eq. (11)</p> <p><math>R_t</math> thermal resistance parameter</p> <p><math>R_t^*</math> free-to-forced-convection parameter</p> <p><math>t</math> ambient temperature</p> <p><math>\Delta t</math> characteristic temperature, Eq. (11)</p> <p><math>T</math> fluid temperature</p> <p><math>u, v</math> velocity components along <math>x</math> and <math>y</math> axes</p>	<p><math>U_\infty</math> ambient velocity</p> <p><math>x, y</math> Cartesian coordinates</p> <p><math>\alpha</math> thermal diffusivity</p> <p><math>\beta</math> coefficient of thermal expansion</p> <p><math>\epsilon</math> infinitesimal number</p> <p><math>\theta</math> dimensionless temperature, Eq. (10)</p> <p><math>\lambda, \lambda^*</math> dummy variables, Eq. (21)</p> <p><math>\nu</math> kinematic viscosity</p> <p><math>\xi, \eta</math> reduced coordinates, Eq. (10)</p> <p><math>\tau</math> designated point for <math>\xi</math></p> <p><math>\phi</math> dependent variable</p> <p><math>\psi</math> stream function</p> <p style="text-align: center;"><b>Subscripts</b></p> <p><math>c</math> denotes cold fluid system</p> <p><math>h</math> denotes hot fluid system</p> <p><math>m</math> integral number of terms in a series</p> <p><math>w</math> condition at the wall</p>
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ever, theoretical studies of this heat transfer process have received very little attention in the literature.

## BASIC EQUATIONS

We consider a vertical flat plate of length  $L$  and thickness  $b$ , which separates two semi-infinite spaces filled with a fluid saturated porous medium as shown in Fig. 1. Far away from the plate the porous medium is considered isothermal, with a temperature  $t_h$  on the right (hot) side of the plate and a temperature  $t_c$  on the left (cold) side. Due to gravity, a free-convection boundary layer appears on the hot side of the plate and flows upward along the plate. We assume that the fluid-saturating porous medium on the cold side of the plate flows downward with velocity  $U_\infty$ . Accordingly, the two fluid streams move in opposite directions. Because of this assumption, the present problem can be formulated in terms of the boundary-layer approximation for two different heat transfer systems. It is also assumed that heat conduction along the plate is neglected in comparison with transverse heat conduction.

Under these assumptions, the boundary-layer equations expressing the conservation of mass, Darcy's law, and the energy of this problem can be written as follows.

Free-convection side:

$$\frac{\partial u_h}{\partial x} + \frac{\partial v_h}{\partial y} = 0 \quad (1)$$

$$u_h = \frac{gK\beta}{\nu}(T_h - t_h) \quad (2)$$

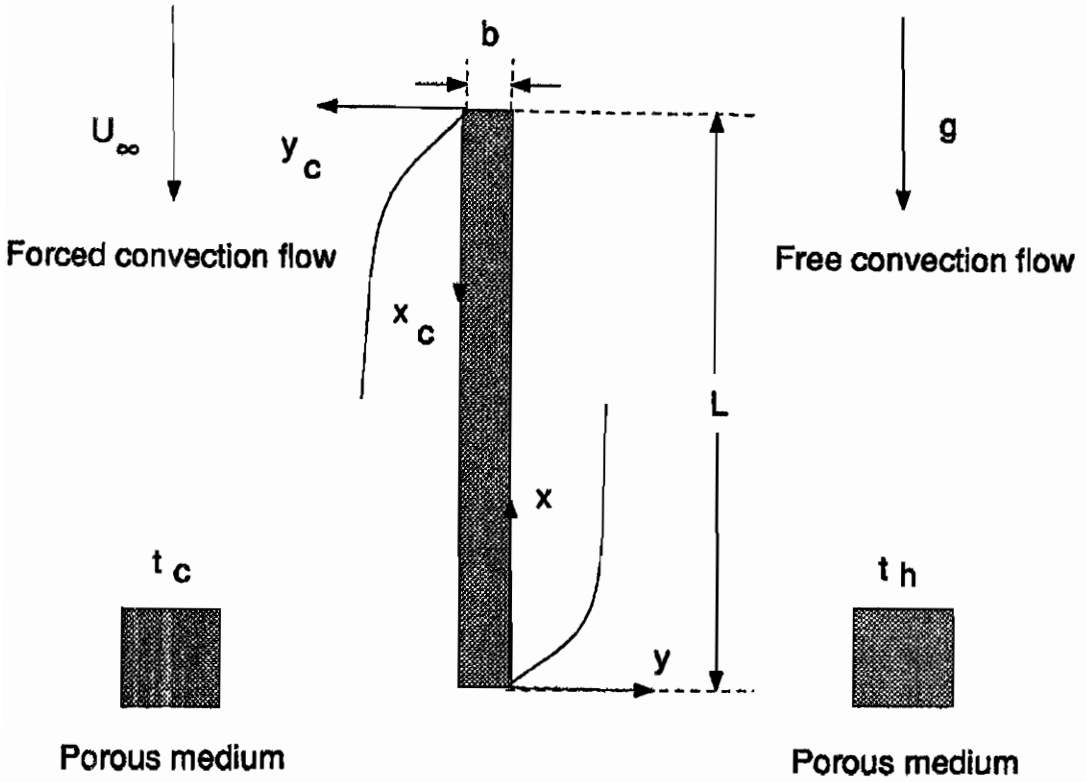


Fig. 1 Schematic diagram of the physical model considered.

$$u_h \frac{\partial T_h}{\partial x} + v_h \frac{\partial T_h}{\partial y} = \alpha_h \frac{\partial^2 T_h}{\partial y^2} \quad (3)$$

where  $x$  and  $y$  are Cartesian coordinates on the hot side of the plate;  $u_h$  and  $v_h$  are the velocity components along  $x$  and  $y$  axes, respectively;  $T_h$  is the fluid temperature of the hot fluid;  $g$  is the acceleration due to gravity;  $K$  is the permeability of the porous medium; and  $\beta$ ,  $\nu$  and  $\alpha_h$  are the coefficients of the thermal expansion, kinematic viscosity, and thermal diffusivity of the porous medium, respectively.

The boundary conditions for the hot system are

$$\begin{aligned} v_h &= 0, & T_h &= T_{wh}(x) & \text{at } y &= 0 \\ u_h &\rightarrow 0, & T_h &\rightarrow t_h & \text{as } y &\rightarrow \infty \end{aligned} \quad (4)$$

where  $T_{wh}(x)$  denotes the wall temperature facing the hot side.

Forced-convection side:

$$u_c = U_\infty \quad v_c = 0 \quad (5)$$

$$U_\infty \frac{\partial T_c}{\partial x_c} = \alpha_c \frac{\partial^2 T_c}{\partial y_c^2} \quad (6)$$

where  $x_c$  and  $y_c$  are Cartesian coordinates on the forced-convection side,  $u_c$  is the velocity along the  $x_c$  axis, and  $T_c$  is the temperature of the cold fluid.

The boundary conditions for the cold system are

$$\begin{aligned} T_c &= T_{wc}(x_c) & \text{at } y_c &= 0 \\ T_c &\rightarrow t_c & \text{as } y_c &\rightarrow \infty \end{aligned} \quad (7)$$

where  $x_c = L - x$  and  $T_{wc}(x_c)$  is the wall temperature facing the forced-convection side. Further, since heat conduction along the plate is neglected in comparison with transverse heat conduction, the heat flux entering the right face of the plate is equal to that leaving the left face at any given position  $x_c$ , that is,

$$-k_s \frac{T_{wc} - T_{wh}}{b} = k \left. \frac{\partial T_h}{\partial y} \right|_{y=0} = -k \left. \frac{\partial T_c}{\partial y_c} \right|_{y_c=0} = q_{x_c} \quad (8)$$

where  $k_s$  and  $k$  denote the thermal conductivities of the plate and of the porous medium, respectively, and  $q_{x_c}$  denotes the local heat flux through the plate. A correlation between  $T_{wh}(x)$  and  $T_{wc}(x_c)$  can be obtained from Eq. (8) as

$$T_{wc}(x_c) = T_{wh}(x_c) - \frac{kb}{k_s} \left. \frac{\partial T_h}{\partial y} \right|_{y=0} \quad (9)$$

## SOLUTION

To solve the boundary-layer equations of two different heat transfer systems described by Eqs. (1)–(3), and (5) and (6), we introduce the following new variables:

$$\begin{aligned} \xi &= \frac{x}{L} & \eta &= \left( \frac{y}{L\xi^{1/2}} \right) Ra^{1/2} \\ \psi_h &= \alpha_h Ra^{1/2} \xi^{1/2} f_h(\xi, \eta) & \theta_h(\xi, \eta) &= \frac{T_h - (t_h + t_c)/2}{\Delta t} \\ \xi_c &= \frac{x_c}{L} = 1 - \xi & \eta_c &= \left( \frac{y_c}{L\xi_c^{1/2}} \right) Pe^{1/2} \\ \theta_c(\xi_c, \eta_c) &= \frac{T_c - (t_h + t_c)/2}{\Delta t} \end{aligned} \quad (10)$$

where  $\psi_h$  is the stream function of the hot system, which is defined in the usual way as  $u_h = \partial\psi_h/\partial y$ , and  $v_h = -\partial\psi_h/\partial x$ . Further,  $Ra$  and  $Pe$  are the Rayleigh and Peclet numbers, and  $\Delta t$  is the heat transfer characteristic defined as

$$Ra = \frac{gK\beta\Delta tL}{\alpha_h\nu} \quad Pe = \frac{U_\infty L}{\alpha_c} \quad \Delta t = t_h - t_c \quad (11)$$

Substituting variables (10) into Eqs. (1)–(3), and (5) and (6), we get

$$\frac{\partial^3 f_h}{\partial \eta^3} + \frac{1}{2} f_h \frac{\partial^2 f_h}{\partial \eta^2} = \xi \left( \frac{\partial f_h}{\partial \eta} \frac{\partial^2 f_h}{\partial \eta \partial \xi} - \frac{\partial^2 f_h}{\partial \eta^2} \frac{\partial f_h}{\partial \xi} \right) \quad (12)$$

$$\frac{\partial^2 \theta_c}{\partial \eta_c^2} + \frac{1}{2} \eta_c \frac{\partial \theta_c}{\partial \eta_c} = \xi_c \frac{\partial \theta_c}{\partial \xi_c} \quad (13)$$

subject to the boundary conditions

$$\begin{aligned} f_h = 0, \quad \frac{\partial f_h}{\partial \eta} = \theta_{wh}(\xi) - 1/2 \quad \text{at} \quad \eta = 0 \\ \frac{\partial f_h}{\partial \eta} \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (14)$$

$$\begin{aligned} \theta_c = \theta_{wc}(\xi_c) = \theta_{wh}(\xi_c) - R_t \xi_c^{-1/2} \frac{\partial^2 f_h}{\partial \eta^2} \Big|_{\eta=0} \quad \text{at} \quad \eta_c = 0 \\ \theta_c \rightarrow -1/2 \quad \text{as} \quad \eta_c \rightarrow \infty \end{aligned} \quad (15)$$

where  $R_t = kbRa^{1/2}/(k_s L)$  is the thermal resistance ratio of the free-convection system to the wall, and  $\theta_{wh}$  and  $\theta_{wc}$  denote the dimensionless wall temperatures facing the free-convection and forced-convection sides, respectively, which are defined as

$$\begin{aligned} \theta_{wh}(\xi) &= \frac{T_{wh} - (t_h + t_c)/2}{\Delta t} \\ \theta_{wc}(\xi_c) &= \frac{T_{wc} - (t_h + t_c)/2}{\Delta t} \end{aligned} \quad (16)$$

Also, using (8) in conjunction with (10) gives

$$R_t^* (\xi_c^{1/2} / \xi_c^{1/2}) \frac{\partial^2 f_h}{\partial \eta^2} \Big|_{\eta=0} + \frac{\partial \theta_c}{\partial \eta_c} \Big|_{\eta_c=0} = 0 \quad \text{at any given position } x_c \quad (17)$$

where  $R_t^* = (Ra/Pe)^{1/2}$  is the free-to-forced-convection parameter. In the work that follows, a comprehensive and noniterative numerical scheme is outlined, which is in contrast to the iterative process proposed by Chen and Chang [11] that uses a guessing strategy. Based on Eq. (17), the boundary conditions can be rewritten as

$$\begin{aligned} f_h &= 0, & \frac{\partial^2 f_h}{\partial \eta^2} &= \xi^{1/2} \lambda^*(\xi) & \text{at } \eta &= 0 \\ \frac{\partial f_h}{\partial \eta} &\rightarrow 0 & & & \text{as } \eta &\rightarrow \infty \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial \theta_c}{\partial \eta_c} &= -R_r^* \xi^{1/2} \lambda^*(\xi) & \text{at } \eta_c &= 0 \\ \theta_c &\rightarrow -1/2 & \text{as } \eta_c &\rightarrow \infty \end{aligned} \quad (19)$$

and

$$\lambda = 0 \quad \text{at} \quad \eta = 0, \eta_c = 0 \quad (20)$$

where the dummy variable  $\lambda$  is defined as

$$\lambda(\xi, \eta) = R_r \lambda^*(\xi) - \frac{\partial f_h}{\partial \eta} + \theta_c - \frac{1}{2} \quad (21)$$

The systems (12), (13), and (21), together with the boundary conditions (18), (19), and (20), can be solved using the following singular perturbation method. The partial differential equations are then reduced to solvable ordinary differential equations only with respect to  $\eta$  (or  $\eta_c$ ), so that the difficulties associated with the singularities at the points  $\xi = 0, 1$  are obviated.

We consider a general form of partial differential equations and their corresponding boundary conditions,

$$G(\xi, \phi_0, \phi_1, \dots, \phi_m, \dots) = 0 \quad (22)$$

where  $\phi_m = \partial^m \phi / \partial \xi^m$ ,  $m = 0, 1, \dots$ , and  $G$  is an operator only with respect to  $\eta$  (or  $\eta_c$ ). Substituting the expansions

$$\xi = \tau + \varepsilon$$

and

$$\phi = \sum_{m=0}^{\infty} \frac{\varepsilon^m}{m!} \phi_m \Big|_{\xi=\tau} \quad (23)$$

into Eq. (22) and then comparing each term of  $\varepsilon^m$  ( $m = 0, 1, \dots$ ) leads to

$$G|_{\xi=\tau} = 0$$

$$\left( \frac{\partial G}{\partial \xi} + \sum_{m=0}^{\infty} \frac{\partial G}{\partial \phi_m} \phi_{m+1} \right) \Big|_{\xi=\tau} = 0$$

and so on. The resulting equations are infinite-dimensional, so in numerical work it is necessary to truncate the Taylor series expansion (23) after a sufficient number of terms for the required accuracy.

**RESULTS AND DISCUSSION**

In this section we present a selection of numerical solutions of Eqs. (12) and (13) subject to the boundary conditions (18)–(20). We shall restrict our attention to those cases for which  $0 \leq \xi \leq 1$  and both the thermal resistance,  $R_p$ , and free-to-forced convection,  $R_f^*$ , parameters have values of 0.5, 1, and 2.

In Figs. 2 and 3 we display the developing profiles of the dimensionless temperature facing the cold plate,  $\theta_{wc}(\xi_c)$ , as  $\xi_c$  increases. It is clear from Fig. 2 that, for increasing values of  $R_p$ , the temperature of the cold side of the plate decreases because the thermal resistance of the free-convection system becomes larger compared with that of the forced-convection flow in the cold system. In contrast, the temperature of the cold side of the plate increases as the free convection becomes dominant, i.e., the parameter  $R_f^*$  is increasing. This can be seen in Fig. 3. This decrease or increase of  $\theta_{wc}(\xi_c)$  leads to the increase of the

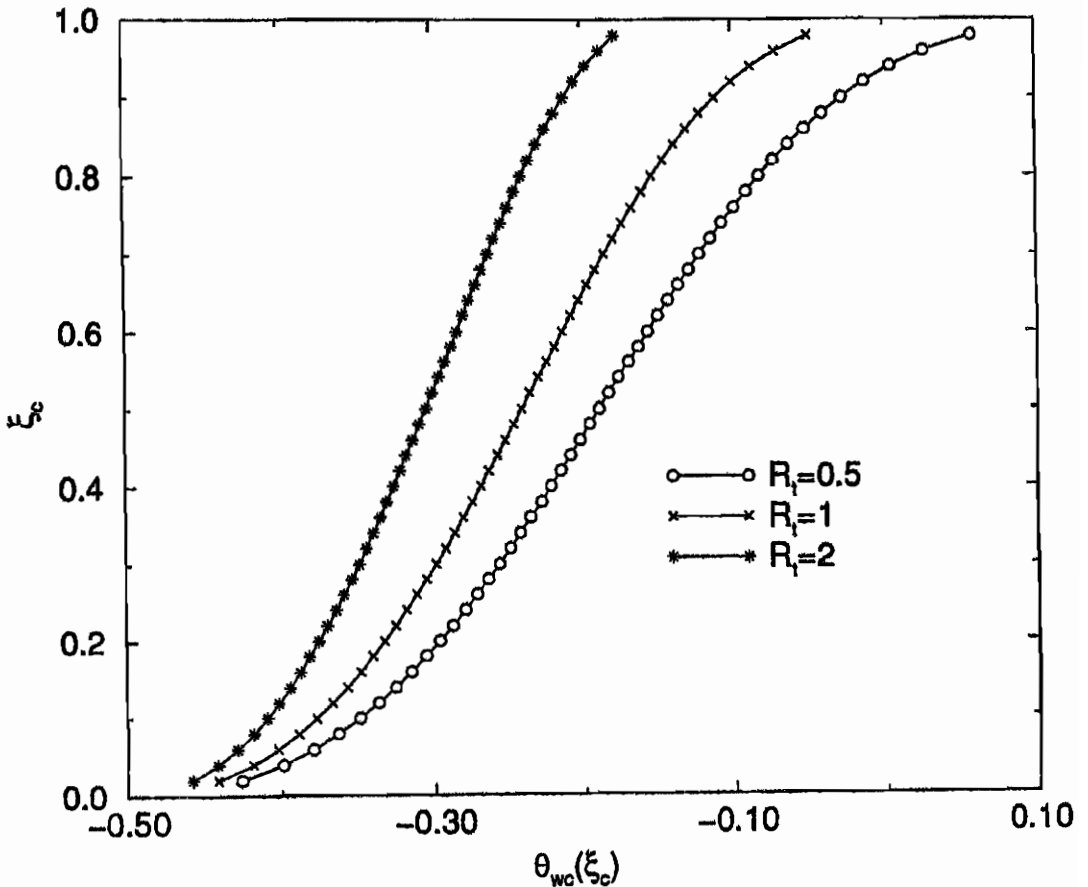


Fig. 2 Effect of  $R_p$  on  $\theta_{wc}(\xi_c)$  for  $R_f^* = 1$ .

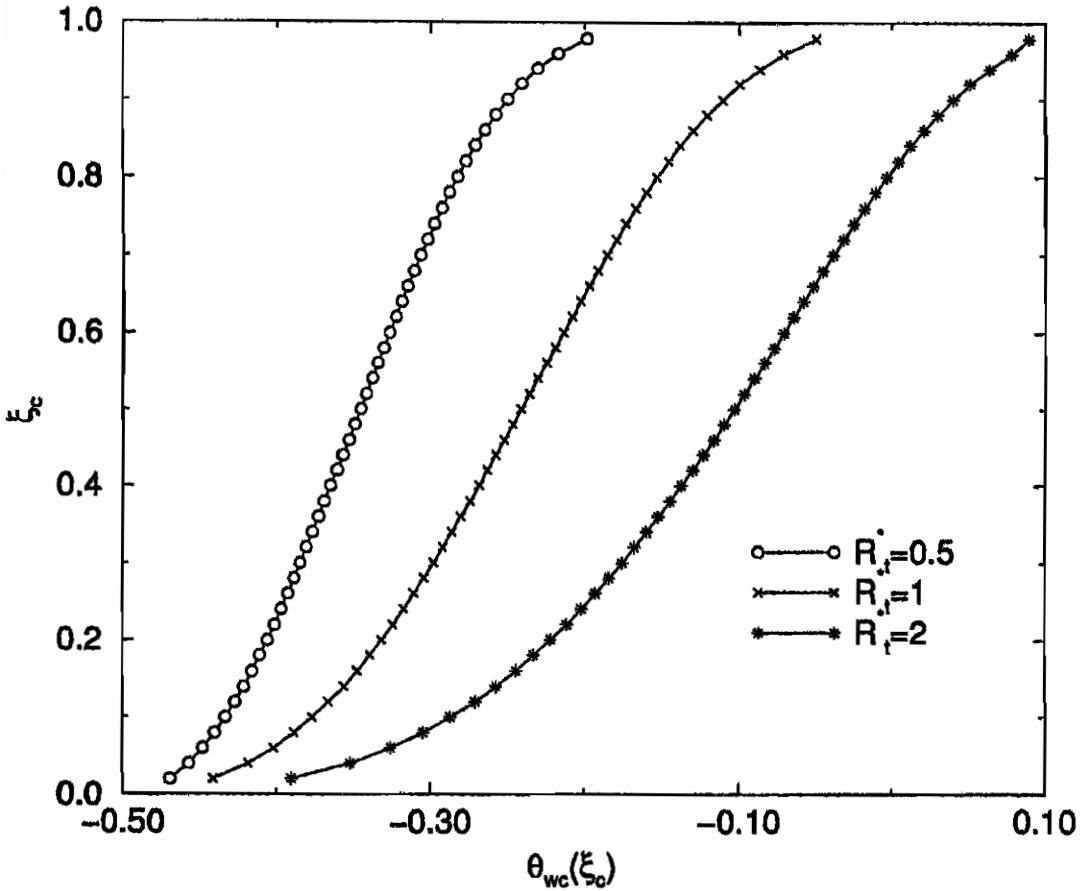


Fig. 3 Effect of  $R_i^*$  on  $\theta_{wc}(\xi_c)$  for  $R_i = 1$ .

heat transfer rate at the cold side of the plate,  $\partial\theta_c(\xi_c, 0)/\partial\eta_c$ , as  $R_i$  increases, and to a reduction of the heat transfer rate on the side facing the forced-convection system with the increase of  $R_i^*$ , as can be seen from Figs. 4 and 5.

Further, Figs. 6 and 7 illustrate the variation of the dimensionless axial velocity at the hot wall,  $\partial f_h(\xi, 0)/\partial\eta$ , as  $\xi$  increases. It is seen from these figures that the axial wall velocity profiles remain negative and increase monotonically with the increasing of the parameters  $R_i$  and  $R_i^*$ . However, the dimensionless skin friction,  $\partial^2 f_h(\xi, 0)/\partial\eta^2$ , is positive for the values of  $R_i$  and  $R_i^*$  considered. Additionally, the skin friction profiles decrease with increasing of both  $R_i$  and  $R_i^*$  parameters, as can be seen from Figs. 8 and 9.

## CONCLUSIONS

In this article, we have studied the free-convection process in contact with one lateral surface of a thin vertical finite flat plate embedded in a porous medium. A forced cooling flow is assumed on the other lateral surface of the plate. The neglect of the heat conduction along the plate allows the transformation of the boundary-layer equations into a dimensionless set of nonsimilar equations of parabolic type, which is then solved using



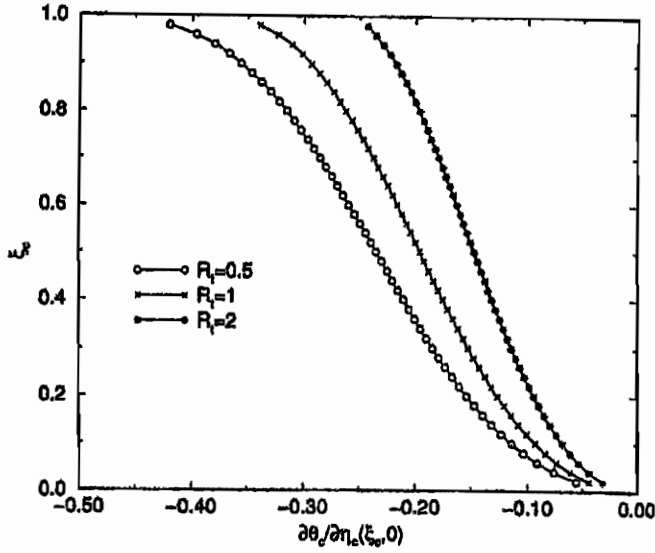


Fig. 4 Effect of  $R_t$  on  $\partial\theta_c(\xi_c, 0)/\partial\eta_c$  for  $R_t^* = 1$ .

a very efficient finite-difference method. It was shown that the effect of the thermal resistance,  $R_t$ , and free-to-forced-convection,  $R_t^*$ , parameters are important and the resulting physical effects in this problem are not negligible but persistent.

Finally, it is worth mentioning that the results reported in the present article are in good agreement with those presented in [7, 11].

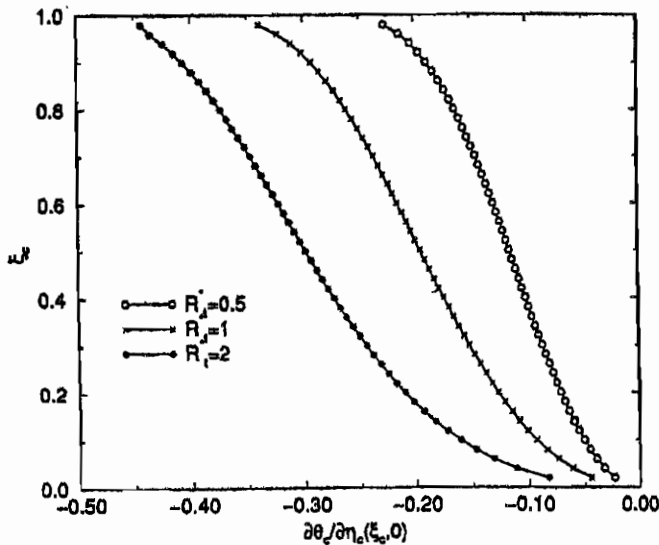


Fig. 5 Effect of  $R_t^*$  on  $\partial\theta_c/\partial\eta_c(\xi_c, 0)$  for  $R_t = 1$ .

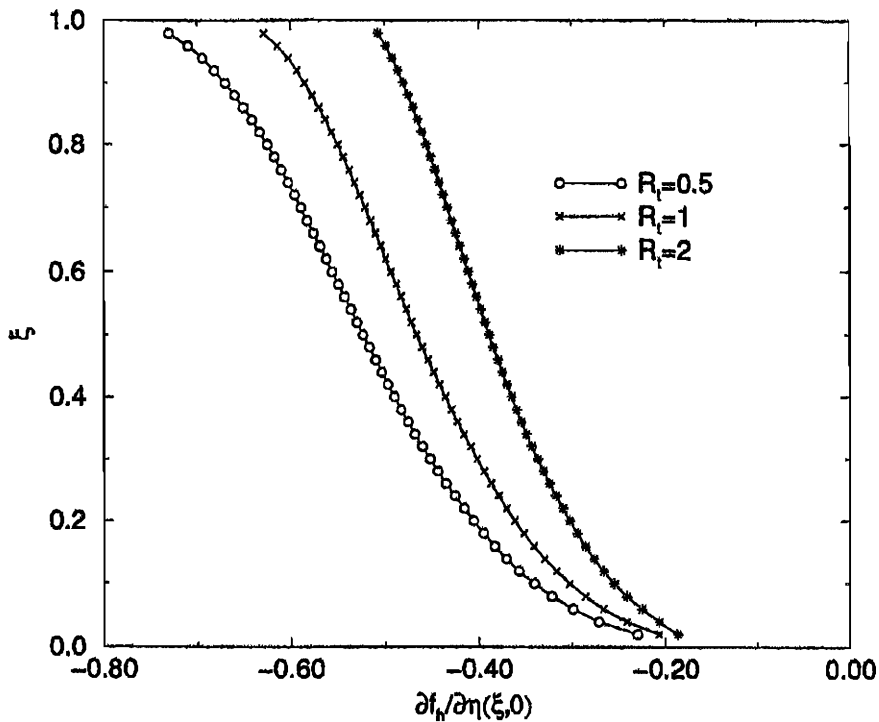


Fig. 6 Effect of  $R_1$  on  $\partial f_1 / \partial \eta(\xi, 0)$  for  $R_1^* = 1$ .

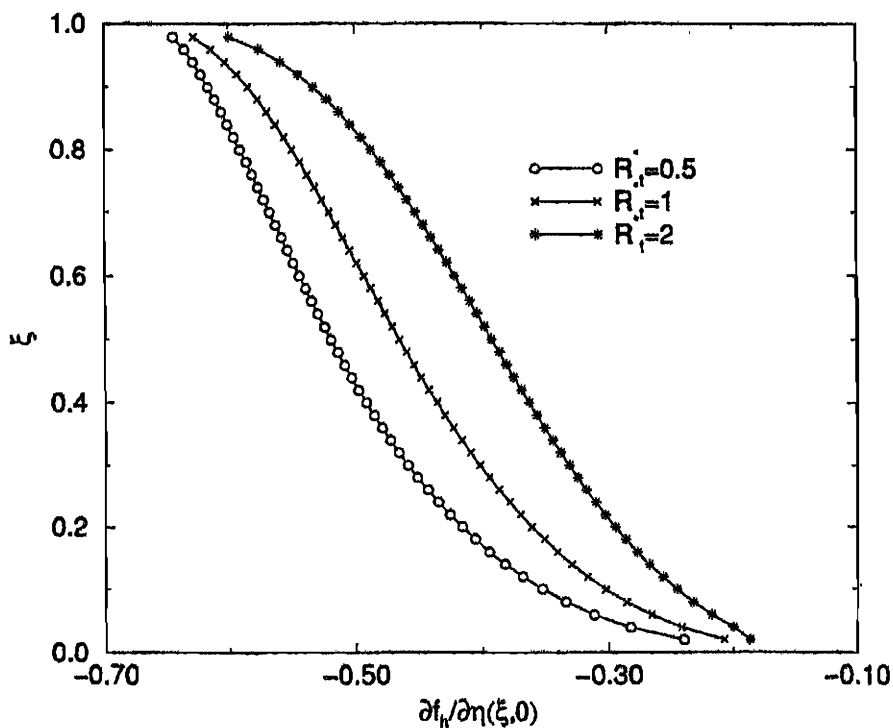


Fig. 7 Effect of  $R_1^*$  on  $\partial f_1 / \partial \eta(\xi, 0)$  for  $R_1 = 1$ .

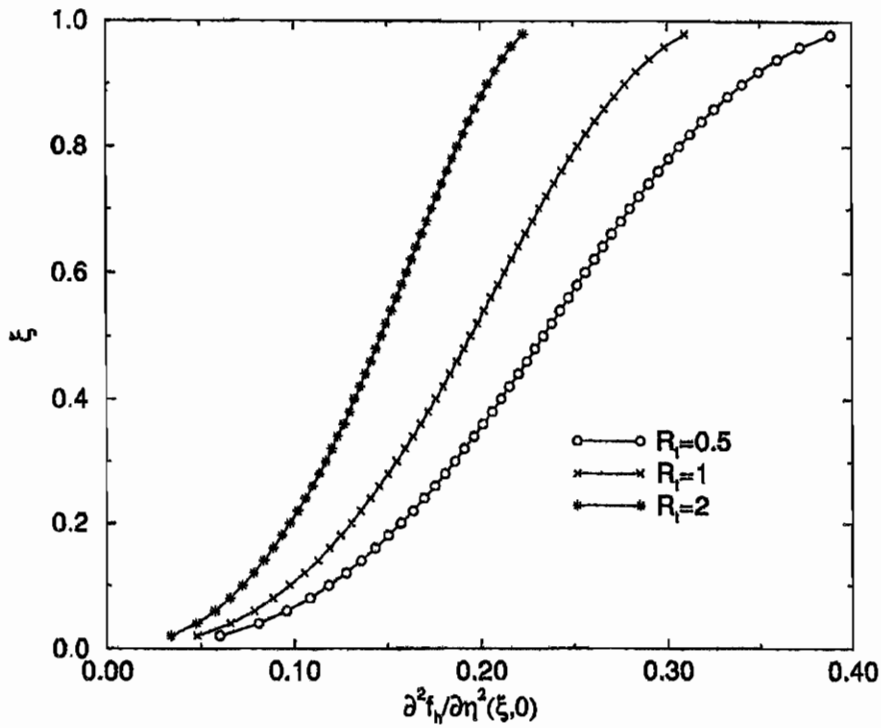


Fig. 8 Effect of  $R_i$  on  $\partial^2 f_w / \partial \eta^2(\xi, 0)$  for  $R_i^* = 1$ .

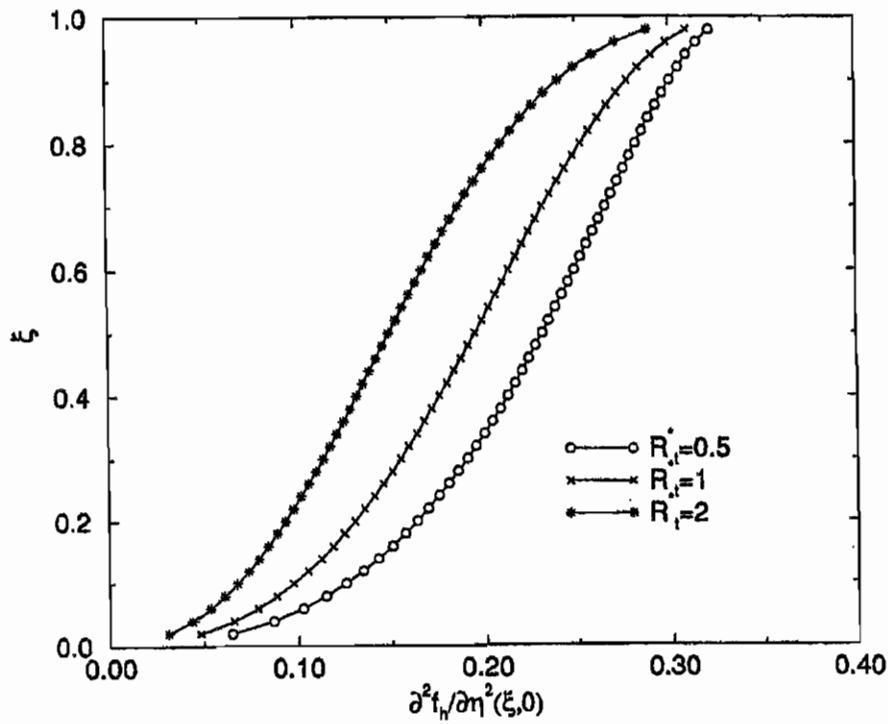


Fig. 9 Effect of  $R_i^*$  on  $\partial^2 f_w / \partial \eta^2(\xi, 0)$  for  $R_i = 1$ .

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